INVISCID HYPERSONIC FLOW PAST PLANE AND AXISYMMETRIC BLUNT BODIES PRODUCING POWER-LAW SHOCKS

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CERTIFICATE

Certified that the thesis 'Inviscid Hypersonic Flow Past Plane and Axisymmetric Blunt Bodies Producing Power-Law Shocks' has been carried out under my guidance and has not been submitted for the award of any degree elsewhere.

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ABSTRACT

This study concerns with the inviscid hypersonic flow over plane and axisymmetric blunt bodies producing a power-law shock. Uniformly valid solutions, far downstream from the blunt nose, have been obtained for the inverse problem, which prescribes the shock leaving the body to be determined, using the method of 'inner and outer expansions' originated by Van Dyke.

The significance of the entropy layer for various power-law shock has been discussed. Power-law bodies for various values of the exponent, n have been constructed for the axisymmetric cas The second-order results for n = 1/2 and 2/3 have been obtained. First-order body and flow variables have been obtained for n = 2/3 in the planar case.

LIST OF SYMBOLS

A	constant, (3.1) and (4.1)
B	constant, $2^{(1/n-1)}/A^{2/n}.n^2$
B_1	constant, $n^2.A^{2/n}$
D	density in the outer region
f	similarity variable for velocity, v
g	similarity variable for density,
h	similarity variable for pressure, p
j	Integer specifying number of dimensions = 0 planar case, = 1 axisymmetric case
n	power-law exponent
n_1	$(1-n)/n$
n_{11}	$(1-n)$
n_2	$2(1-n)$
P	pressure in the outer region
p	pressure in the inner region
$\vec{q}$	velocity vector
R	density in the outer region
t	time
U	velocity in the outer region parallel to the x-direction
u	velocity in the inner region parallel to the x-direction

V	velocity in the outer region parallel to the y-direction
v	velocity in the inner region parallel to the y-direction
(X,x)	coordinates along the body axis
(Y,y)	coordinates in the lateral direction
$\bar{Y}^*$	proportional distance across the shock layer
ρ	density in the inner region
λ	ratio of specific heats
ψ	stream function
σ	local shock angle
M	mach number

Subscripts:

b	evaluated at body surface
1	first order
2	second order
∞	free stream conditions

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FIGURE NO.

CHAPTER 1

INTRODUCTION

Hypersonic flow past blunt bodies with shock waves of very great intensity that have a power-law form are considered for the plane and axi-symmetric cases. For the power-law exponent, $n < 1$, the shock shape being blunt near the nose, the stream lines passing through it are subjected to large changes in entropy resulting in the formation of a layer of relatively high entropy gas. This layer sometimes referred to as the 'Entropy Layer' exhibits flow properties quite similar to a boundary layer and is different from the slightly disturbed outer flow field. The influence of this entropy layer on the asymptotic flow field has been analysed by Sychev[3], Yakura[7] and several others. The outer region where the streamlines cross the oblique portion of the shock-wave can be treated by hypersonic slender body theory (1). Sychev has considered the inverse problem of determining the body that produces a power-law shock-wave specifying a paraboloidal shock-wave and properly accounting for the entropy layer he has shown a method to construct the body shape numerically. Yakura[7] has considered the inverse problem using the method of inner and outer expansion (8) to obtain uniformly valid solutions far downstream from the nose of a slightly blunted body in hypersonic flow. An attempt has been made in this work to construct analytical

solutions for the inverse problem, prescribing a power-law shock-wave. The significance of the entropy layer for various power-law exponents have been discussed and power-law bodies constructed.

For the slender bodies the line of development has been concerned with the small disturbance theory, where it is assumed that the velocity perturbations are small compared with the free stream velocity. The velocity perturbations, however, are not small compared with the local sound velocity and the pressure perturbations are not small compared with the free stream static pressure. Hence the disturbances are not at all small in the sense usually associated with the linearised supersonic theory. This theory started with the hypersonic similitude principle. The idea is due to Tsien (1), though Hayes introduced the general hypersonic similitude principle which states that a small disturbance hypersonic flow is equivalent to an unsteady flow in one less space dimension. Particular group of solutions to the small disturbance equations that have been worked out have been almost exclusively of the self-similar type. They are generally termed similar solutions, and have the property that the solution in the lateral variable y or r at one value of t (or of x for the corresponding hypersonic steady flow) is similar to the solution at any other value of t. This property permits a decrease in the number of independent variables from two to one thereby reducing the partial

differential equation to the ordinary one, the treatment of which is much easier. Many of the investigations of self similar solutions of the blast wave type are directly applicable to the hypersonic steady flow problems. Lees and Kubota have used the blast wave theory for simulating the flow past blunt bodies. But the analogy between the flow field of an intense explosion and the steady transverse flow field of a blunt body moving hypersonically in one more space dimension is a small disturbance approximation valid only far from the body. The strong perturbations due to the blunted nose creates a region of high entropy gas near the body in which the small disturbance equations and the hypersonic similitude principle fail to apply. The entropy layer is similar to the boundary layer, in that the pressure is essentially constant across it and the velocity gradients are large. The entropy layer also has a displacement effect on the flow, like the boundary layer.

Sychev[3] considers the inverse problem of determining the body that produces the outer small-disturbance flow field given by the analogous blast wave. He recognizes the shortcomings of the blast wave theory, and for the case of plane and axisymmetric gas flow with shock waves of power-law form, he shows that the use in the flow problem of the exact solution for the corresponding unsteady self similar gas motion requires a supplementary refinement of the thickness of the high entropy

layer. He also shows a method for introducing such a correction and constructing the shape of the body contour on which is to be applied the pressure distribution on the basis of the theory of small disturbances. From his study he concludes that: i) the 'equivalence principle' or the law of plane sections is not valid in the entropy layer. ii) The blast wave analogy applies to a specific body that has a relatively large lateral dimensions compared with the nose dimensions. iii) In the range of values of γ of practical interest there exists a definite interval of values of the exponent n, $2/3+j \leq n \leq n^*$ ($j = 0$ or 1 for planar or axi-symmetric cases and $n^* = (2 + 2/\gamma) / (3 + \gamma + 2/\gamma)$) where proper consideration of the entropy layer is necessary for e.g. in determining the body contour for a given shape of the shock-wave. Specifying a parabolic shock-wave and properly accounting for the entropy layer, he calculates a body shape that becomes very large far downstream. And finally he concludes that the self similar motions in all cases certainly retain their significance as asymptotic representations of the exact solution.

It was first shown by Guiraud in his study of the flow past a blunt flat plate that the entropy layer is a region of non-uniformity, characteristic of a singular perturbation problem and can therefore be treated by a perturbation method first applied to the boundary layers. Guiraud[9] concludes that

the blast wave theory is correct as far as the leading term is concerned, the entropy layer being felt only through the first correction, thereby settling the point which has been controversial in the past namely, whether the displacement effect of the entropy layer generated by the detached shock, ahead of the nose, invalidates the blast wave analogy theory.

Yakura uses the method of inner and outer expansion in obtaining uniformly valid solutions far downstream from the nose of a slightly blunted body in hypersonic flow. This method, originated by M.D. Van Dyke[8] is applied to the inverse problem. He transforms the equation of motion such that the outer region of the shock layer and the inner region corresponding to the entropy layer can be treated separately. With a proper choice of variables he forms the asymptotic expansions for each region. Although real gas effects become significant at hypersonic speeds he restricts the analysis to the case of an inviscid, non heat conducting perfect gas, and to the limiting case of Mach number tending to infinity. He treats the plane and axisymmetric flows past bodies producing hyperbolic and power-law shocks The hyperbolic shocks correspond to blunted wedges and cones in two and three dimensions respectively. The exponents of the power-law shocks considered are $n = 1/2$ for axi-symmetric case and $n = 2/3$ for the plane case. Yakura concludes that the entropy layer is, in many respects, analogous to Prandtl's

viscous boundary layer with common features of displacement thickness, zero normal pressure gradient, and a velocity gradient. The flow field influenced by the entropy layer varies with body or shock geometry. When $n = 1$ (wedge or cone) the effects of bluntness become (for large x) concentrated near the surface; whereas for $n = 1/2$ they spread out into the flow field having the same rate of growth as the body. Thus, the relative thickness of the entropy layer (compared with the body radius) becomes appreciable only for values of n much less than unity.

Presented in this study are analytical solutions of the flow field in the form of asymptotic expansions uniformly valid for large distances downstream. The examples treated in this study are plane and axi-symmetric flows past power-law shocks of any exponent $n (2/3+j < n < 1)$. Yakura's method is extended to the general case of power-law shocks. Here again the equations of motion are transformed such that the outer region of the shock layer and inner entropy layer can be treated separately. With a proper choice of variables, asymptotic expansions are formed for each region. The outer limit process transfers the region of non-uniformity (entropy layer) to the inner unknown boundary (body) and the outer expansion formed is valid every where except at the inner boundary. The inner limit process transfers the outer region to infinity

and the inner expansion is valid in the inner region but not far away. The limiting process is equivalent to shrinking the co-ordinate system or physically moving to infinity in the downstream direction with the nose bluntness fixed. It is the inner expansion which determines the body. From a physical view point, the outer limit consists in moving far downstream (or, alternatively letting the nose radius shrink to zero) while remaining away from the surface, whereas in the corresponding inner limit one stays within some multiple of the nose radius of the surface. Boundary conditions for the outer problem are the shock conditions and for the inner problem use can be made of matching between the inner and outer expansions to extract the inner boundary conditions. A uniformly valid solution is then constructed by forming a composite expansion consisting of both the inner and outer expansions.

CHAPTER II

GENERAL ANALYSIS

Flow Equations:

Steady flow of an inviscid perfect gas with constant specific heats is described by the following set of equations.

$$\operatorname{div}\ (\rho \vec{q}) = 0 \tag{2.1a}$$

$$\rho\,(\vec{q} \cdot \operatorname{grad})\vec{q} + \operatorname{grad}\ p = 0 \tag{2.1b}$$

$$\vec{q} \cdot \operatorname{grad}\ (p/\rho^{\gamma}) = 0 \tag{2.1c}$$

The conservation equations written in 2-dimensional cartesian co-ordinates are

$$\frac{\partial}{\partial x}\,(\rho u) + \frac{\partial}{\partial y}\,(\rho v) = 0 \tag{2.2a}$$

$$\rho\,(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}) + \frac{\partial p}{\partial x} = 0 \tag{2.2b}$$

$$\rho\,(u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}) + \frac{\partial p}{\partial y} = 0 \tag{2.2c}$$

$$u\,\frac{\partial}{\partial x}\,(p/\rho^{\gamma}) + v\,\frac{\partial}{\partial y}\,(p/\rho^{\gamma}) = 0 \tag{2.2d}$$

Equation (2.2d) expresses the fact that the entropy is constant along streamlines between the shockwaves and

the body. This property has been exploited by defining a stream function that satisfies the continuity equation as an indepen-variable, using Von-Mises transformation. Then the differential equations (2.2a - e) become

$$\frac{\partial y}{\partial x} = \frac{v}{u} \tag{2.3a}$$

$$y^j \frac{\partial y}{\partial \psi} = \frac{1}{\rho u} \tag{2.3b}$$

$$\frac{\partial y}{\partial x} + y^j \frac{\partial p}{\partial \psi} = 0 \tag{2.3c}$$

$$u^2 + v^2 + \frac{2\gamma}{\gamma - 1} p/\rho = 1 \tag{2.3d}$$

$$p/\rho^\gamma = f(\psi) \tag{2.3e}$$

where $j = 0$ for plane flow and $j = 1$ for axi-symmetric flow.

Boundary Conditions:

Flow variables have been made dimensionless by referring velocities and density to their free stream values u_∞ and ρ_∞ and pressure to $\rho_\infty u_\infty^2$. Equations (2.3a-e) are unchanged with respect to this transformation. Boundary conditions are the flow quantities behind the shock-wave and are obtained from the oblique shock relations and expressed

in the form for $M = \infty$

$$\frac{p}{p_\infty} = 1 + \gamma M_\infty^2 \sin^2\sigma \, (1 - \epsilon)$$

where $\epsilon = \frac{\gamma - 1}{\gamma + 1}$

$$M_\infty^2 = \frac{u_\infty^2}{a_\infty^2} = \frac{u_\infty^2 \, \rho_\infty}{p_\infty \, \gamma}$$

Dividing throughout by γM_∞^2 we get

$$\frac{p}{\rho_\infty \, u_\infty^2} = \frac{1}{\gamma M_\infty^2} + \frac{2}{\gamma + 1} \sin^2\sigma$$

$M_\infty \rightarrow \infty$ we get the dimensionless pressure as

$$p = \frac{2}{\gamma + 1} \sin^2\sigma$$

$$\frac{\rho}{\rho_\infty} = \frac{\gamma + 1}{\gamma - 1}$$ or in the dimensionless form

$$\rho = \frac{\gamma + 1}{\gamma - 1}$$

$$\bar{u}' = \cos\sigma$$

$$\bar{v}' = \epsilon \sin\sigma$$

where $\bar{u}'$ and $\bar{v}'$ are the velocities tangential and normal to the

shock and σ is the local shock angle.

$$u = \cos^2\sigma + \frac{\gamma-1}{\gamma+1}\sin^2\sigma$$

$$= 1 - \frac{2}{\gamma+1}\sin^2\sigma$$

$$v = \cos\sigma\sin\sigma - \frac{\gamma-1}{\gamma+1}\cos\sigma\sin\sigma$$

$$= \frac{2}{\gamma+1}\cos\sigma\sin\sigma$$

$$y = [\,(1+j)\,\psi\,]^{1/(1+j)}$$

Boundary conditions at the shock are

$$p = \frac{2}{\gamma+1}\sin^2\sigma \tag{2.4a}$$

$$\rho = \frac{\gamma+1}{\gamma-1} \tag{2.4b}$$

$$u = 1 - \frac{2}{\gamma+1}\sin^2\sigma \tag{2.4c}$$

$$v = \frac{2}{\gamma+1}\sin\sigma\cos\sigma \tag{2.4d}$$

$$y = [\,(i+j)\,\psi\,]^{1/(1+j)} \tag{2.4e}$$

Shock-Wave

The blunt shock-wave is taken to have an analytic functional form

$$y = Ax^n$$

y is the normal co-ordinate with respect to the free stream direction x, in both plane and axi-symmetric flows. The local shock angle is given by

$$\tan \sigma = An\, x^{n-1}$$

The form of the stream function at the shock is given as

$$\psi = \frac{(Ax^n)^{(1+j)}}{1+j} \tag{2.5}$$

Equations (2.3a-e) are the set of five equations in the five unknowns, (p, ρ, u, v and y) as functions of the two independent variables x and ψ subject the shock conditions (2.4a-e). Equation (2.3e) gives an insight into the extent of the entropy layer. The entropy function p/ρ^{γ} on any stream-line depends only upon the local shock angle evaluated at the point of intersection of the shock and the stream-line. The streamlines which intersect the shock far downstream and far out from the body define an outer region where the stream-line is of the order of $x^{n(i+j)}$. In this region the entropy

function takes on its small disturbance value or approaches a uniform value depending on whether $n < 1$ or $n = 1$. The region in which ψ is of the order of 1 corresponds to the entropy layer. In this 'inner region' the stream-lines originate in the vicinity of the nose and p/ρ^{γ} is relatively large. The shock is assumed to be smooth and so the entropy is a continuous function of the stream function. For thorough investigation of the two regions of interest the method of inner and outer expansions have been used.

Inner and Outer Expansions

Asymptotic expansion for large down-stream distances have been developed. The small perturbation parameter associated with the limits and expansion is $1/x$ where x is the longitudinal down-stream co-ordinate as shown in Fig. 1. A new independent variable of order unity is introduced in the outer-region extending to the shock.

Outer Independent Variables

$$X = x$$
$$w = \psi/x^{n(1+j)}$$

Outer Dependent Variables

P, D, U, V, Y which are respectively the

pressure, density, velocities along and perpendicular to the X-direction and the normal co-ordinate.

This choice of the outer independent variable is fully consistent with the definition of the outer region, where ψ is of the order of $X^{n(1+j)}$.

Let $P(X,w)$ denote pressure in the outer region or in general let $F(X,w)$ be any flow quantity in the outer region. Outer limit is defined as the limit as the perturbation parameter tends to zero with the outer variables fixed i.e.

$$O \lim [F(X,w)] = \lim F(X,w)$$

as $X \to \infty$ keeping w fixed.

In the outer region w is of the order one. In the inner region ψ is of the order one and the inner independent variables are x and ψ and the dependent variables are p, ρ, u, v and y. The inner limit is defined as the limit of a flow quantity as $x \to \infty$ keeping ψ fixed. Let $f(x,\psi)$ denote a flow quantity

$$I \lim [\, f(x,\psi) \,] = \lim f(x, \psi)$$

as $x \to \infty$ keeping ψ fixed.

The outer limit consists in moving away from the surface some appreciable fraction of the distance to the shock and the inner limit considers the region near the surface.

The equations of Section 2.3 are transformed into the new inner any outer variables. The 'outer equations' which are functions of (X,w) are valid in the outer region and they will be used to determine the outer small disturbance solutions. For the outer equations transformed shock conditions are the boundary conditions. The inner equations are dependent on (x,ψ) and are restricted to the inner region and these determine the inner solution taking care of the displacement effect of the entropy layer. Inner equationsas a result of the asymptotic expansion suffer a loss of boundary conditions.

Composite Expansion:

Composite expansion is constructed from the inner and outer expansion and is valid in the entire flow field. Both the inner and outer expansions are not valid in the regions of their counterpart. The sum of the inner and outer expansions are taken, outer expansion being rewritten in the inner variables and the outer expansion of the inner expansion is subtracted to remove duplication. The composite expansion takes the form

$$\bar{f}(x,\psi) = \mathrm{I\ ex}[\ f(x,\psi)\ ] + \mathrm{O\ ex}[\ F(x,\psi)\ ] - \mathrm{O\ ex}\ [\ \mathrm{I\ ex}[\ f(x,\psi)\ ]\ ]$$

Near the body, as $w \to 0$, contributions of the outer expansions cancel out.

Shock-Wave of the Power-Law Form

As already stated the theory of small disturbances in a hypersonic stream, the problem of flow past a plane or axisymmetric shock-wave or body is equivalent to the problem of one-dimensional unsteady gas motion under the action of a plane or cylindrical piston. In this analogy the class of self similar motions with very intense shock-waves propagating according to a power-law corresponds to a class of steady flows with shock-waves of a power law form $y = Ax^n$. With the mach number of the undisturbed stream $M_\infty \to \infty$ values of the exponent lying in the interval $2/3+j < n < 1$, where $j = 0$ or 1, correspond to flows past convex bodies of power-law form. The case $n = 2/3+j$ is singular and corresponds to the problem of a strong explosion. In this case the interpretation as a flow problem consists in the assumption of a finite drag force acting on the leading edge of a body of vanishing thickness. In other words there is an analogy between the appearance of a strong explosion and the effect of blunting the leading edge of a slender body at large distances from the bluntness.

At sufficiently large distances from the leading edge of the body the transverse velocity component v is proportional to the local angle of inclination of the shock surface 'σ' and the pressure to the square of the sine of the angle.

$$v \sim U_\infty \sigma \quad , \quad p \sim \rho_\infty U_\infty^2 \sigma^2 \quad , \quad \sigma \sim \frac{A}{x^{1-n}}$$

Substituting these estimates in equation (2.3c) and (2.3b) we find that the relative change in pressure across the entropy layer is

$$\frac{\Delta p}{p} \sim \sigma^{n(1+j)/(1-n)}$$

$$\therefore \quad \frac{\Delta p}{p} \leq \sigma^2 \quad \text{for } n \geq 2/3+j$$

Since the small disturbance theory[5] involves just the same limit of accuracy the change in pressure can be neglected. For the estimate of the relative thickness of the entropy layer Equations (2.3e) and (2.3b) are used. From the condition of constancy of the entropy along stream-lines and the boundary conditions for p and ρ we find that along the entire entropy layer

$$\frac{p}{\rho^\gamma} \sim \frac{\rho_\infty U_\infty^2}{\left[\frac{(\gamma+1)}{(\gamma-1)} \rho_\infty\right]^\gamma}$$

Using the estimate for the pressure p, we obtain the following estimate for ρ

$$\rho = K \rho_\infty \sigma^{-2/\gamma}$$

Taking $u \sim U_\infty$ using (2.3b) we find that relative thickness of

the entropy layer is

$$\frac{\Delta y}{\gamma} \sim \frac{1}{\frac{(\gamma+1)}{(\gamma-1)}} \sigma^{\,n(1+j)/(1-n) - 2/\gamma}$$

It is negligibly small when the exponent of σ is greater than 2 i.e.

$$n(1+j)/(1-n) - \frac{2}{\gamma} \geq 2$$

$$n \geq n^{*} = (2+2/\gamma)/(3+j+2/\gamma)$$

This means that in the range of values of γ of practical interest there exists a definite interval of values of the exponent n, $2/3+j \leq n \leq n^{*}$ where proper consideration of the entropy layer is necessary for e.g. in determining the body contour for a given shape of shock-wave. An attempt has been made in this present work to find out the effect of entropy layer for various n.

CHAPTER III

BLUNT AXISYMMETRIC CASE

The two different cases namely the axisymmetric and plane flows give rise to different boundary conditions and hence different outer expansion. Therefore the two cases are treated separately.

The shock in Fig. (1) is given by

$$Y = A\,x^{n} \tag{3.1}$$

From equation (2.5) the form of the stream function is

$$\psi = [\,Ax^{n}\,]^{2}/2 \tag{3.2}$$

The basic equations for axisymmetric case are

$$\frac{\partial y}{\partial x} = \frac{v}{u} \tag{3.3a}$$

$$y\,\frac{\partial y}{\partial \psi} = \frac{1}{\rho u} \tag{3.3b}$$

$$\frac{\partial v}{\partial x} + y\,\frac{\partial p}{\partial \psi} = 0 \tag{3.3c}$$

$$u^{2} + v^{2} + \frac{2\gamma}{\gamma-1}\,\frac{p}{\rho} = 1 \tag{3.3d}$$

$$\frac{p}{\rho^{\gamma}} = \left(\frac{\gamma-1}{\gamma+1}\right)\frac{2}{\gamma+1}\,\sin^{2}\sigma \tag{3.3e}$$

$\sin\sigma$ and $\cos\sigma$ are found in terms of stream function ψ as

$$\sin^2\sigma = \frac{1}{1+B\psi^{n_1}} \tag{3.4a}$$

$$\cos^2\sigma = \frac{B\psi^{n_1}}{1+B\psi^{n_1}} \tag{3.4b}$$

where $n_1 = \frac{1-n}{n}$ and

$$B = 2^{(1/n\,-1)} / A^{2/n}\, n^2$$

Boundary conditions follow from (2.4)

$$p = 2/(\gamma+1)\ (1+B\psi^{n_1}) \tag{3.5a}$$

$$\rho = (\gamma+1)/(\gamma-1) \tag{3.5b}$$

$$u = 1 - 2/(\gamma+1)(1+B\psi^{n_1}) \tag{3.5c}$$

$$v = \frac{2}{\gamma+1}\ \frac{\sqrt{B\psi^{n_1}}}{1+B\psi^{n_1}} \tag{3.5d}$$

$$y = \sqrt{2\psi} \tag{3.5e}$$

Outer Expansions

The outer variables are chosen as (X,w) where

$$X = x \tag{3.6a}$$

$$w = \psi/(x^n)^2 \tag{3.6b}$$

The outer expansion is valid in the region away from the body and w is so chosen to be of the order one so that the outer variables are expanded in powers of x for large x distances.

Equations (3.3a-e) are transformed into the following equations with P, D, U, V and Y as the dependent variables

$$Y \frac{\partial Y}{\partial X} = \frac{YV}{U} + 2n\, X^{2n-1} \frac{w}{DU} \qquad (3.7a)$$

$$Y \frac{\partial Y}{\partial w} = \frac{1}{DU} X^{2n} \qquad (3.7b)$$

$$X^{2n} \frac{\partial V}{\partial X} - 2n\, X^{2n-1}\, w \frac{\partial V}{\partial w} + Y \frac{\partial p}{\partial w} = 0 \qquad (3.7c)$$

$$U^2 + V^2 + \frac{2\gamma}{\gamma-1} \frac{P}{D} = 1 \qquad (3.7d)$$

$$\frac{P}{D^\gamma} = \left(\frac{\gamma-1}{\gamma+1}\right)^\gamma \frac{2}{\gamma+1} \frac{1}{1+B(wX^{2n})^{n_1}} \qquad (3.7e)$$

The shock conditions are transformed into the following

$$P = \frac{2}{\gamma+1} \frac{1}{1+B(wX^{2n})^{n_1}}$$

$$= \frac{2}{\gamma+1} \frac{1}{w^{n_1} BX^{n_2}} \left[1 - \frac{1}{Bw^{n_1} X^{n_2}} + \cdots\cdots \right] \qquad (3.8a)$$

$$D = \frac{\gamma+1}{\gamma-1} \tag{3.8b}$$

$$U = 1 - \frac{2}{\gamma+1} \frac{1}{w^{n_1} B X^{n_2}} \left[1 - \frac{1}{B w^{n_1} X^{n_2}} + \ldots\ldots \right] \tag{3.8c}$$

$$V = \frac{2}{\gamma+1} \frac{1}{\sqrt{B}\, w^{n_1/2} X^{n_2/2}} \left[1 - \frac{1}{B w^{n_1} X^{n_2}} + \ldots \right] \tag{3.8d}$$

$$Y = \sqrt{2wX^{2n}} \tag{3.8e}$$

at $\qquad w = A^2/2$

where $\quad n_2 = 2n\, n_1$

The asymptotic expansions for the outer flow quantities are the following, which can be arrived at from equations (3.8a-e)

$$P(X,w) = \frac{P_1(w)}{X^{n_2}} + \frac{P_2(w)}{X^{2n_2}} + \ldots \tag{3.9a}$$

$$D(X,w) = D_1(w) + \frac{D_2(w)}{X^{n_2}} + \ldots \tag{3.9b}$$

$$U(X,w) = 1 + \frac{U_1(w)}{X^{n_2}} + \frac{U_2(w)}{X^{2n_2}} + \ldots \tag{3.9c}$$

$$V(X,w) = \frac{V_1(w)}{X^{n_2/2}} + \frac{V_2(w)}{X^{3n_2/2}} + \ldots \tag{3.9d}$$

$$Y(X,w) = Y_1(w)\,X^n + \frac{Y_2(w)}{X^{n_2-n}} + \dots \tag{3.9e}$$

Substituting the above expansions into equations (3.7a-e) we get the following equations.

First Outer Equation:

$$Y_1 - \frac{2w}{D_1Y_1} = \frac{V_1}{n} \tag{3.10a}$$

$$Y_1\frac{dy_1}{dw} = \frac{1}{D_1} \tag{3.10b}$$

$$V_1 + \frac{2w}{n_1}\frac{dV_1}{dw} - \frac{2}{n_2}Y_1\frac{dP_1}{dw} = 0 \tag{3.10c}$$

$$\frac{P_1}{D_1^{\gamma}} = \left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma}\frac{2}{\gamma+1}\,\frac{1}{Bw^{n_1}} \tag{3.10d}$$

$$2U_1 + V_1^2 + \frac{2\gamma}{\gamma-1}\frac{P_1}{D_1} = 0 \tag{3.10e}$$

Boundary conditions

$$P_1 = \frac{2}{\gamma+1}A^2n^2 \tag{3.11a}$$

$$D_1 = \frac{\gamma+1}{\gamma-1} \tag{3.11b}$$

$$U_1 = -\frac{2}{\gamma+1}A^2n^2 \tag{3.11c}$$

$$V_1 = -\frac{2}{\gamma+1}An \tag{3.11d}$$

$$Y_1 = A \tag{3.11e}$$

Second Outer Equations:

$$nY_1Y_2 + (n-n_2)\,Y_1Y_2 = -Y_1V_1U_1 + Y_2V_1 + Y_1V_2 + 2nw\left[-\frac{U_1}{D_1} - \frac{D_2}{D_1^{\,2}}\right] \tag{3.12a}$$

$$Y_1\frac{dY_2}{dw} + Y_2\frac{dY_1}{dw} = -\frac{D_2}{D_1^{\,2}} - \frac{U_1}{D_1} \tag{3.12b}$$

$$V_2 + \frac{2w}{3n_1}\frac{dV_2}{dw} - \frac{2}{3n_2}\left[Y_1\frac{dP_2}{dw} + Y_2\frac{dP_1}{dw}\right] = 0 \tag{3.12c}$$

$$2U_2 + U_1^{\,2} + 2V_1V_2 + \frac{2\gamma}{\gamma-1}\left[\frac{P_2}{D_1} - \frac{P_1D_2}{D_1^{\,2}}\right] = 0 \tag{3.12e}$$

$$\frac{P_2}{D_1^{\gamma}} - \frac{P_1D_2}{D_1^{\,\gamma+1}} = \left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma}\frac{2}{\gamma+1}\,\frac{1}{B^2w^{2n_1}} \tag{3.12d}$$

Boundary conditions:

$$P_2 = -\frac{2}{\gamma+1}A^4n^4 \tag{3.13a}$$

$$D_2 = 0 \tag{3.13b}$$

$$U_2 = + \frac{2}{\gamma+1} A^4 n^4 \qquad (3.13c)$$

$$V_2 = - \frac{2}{\gamma+1} A^3 n^3 \qquad (3.13d)$$

$$Y_2 = 0 \qquad (3.13e)$$

Solution of the Outer Equation:

Equations (3.10a,b,c and d) are to be solved for the unknowns P, D, V, and Y and U can be calculated from equation (3.10e). Two different methods have been adopted for the solution of the first outer equation, namely, the similarity approach and the numerical integration by the Runge-Kutta method.

Similarity Approach:

The results of integration of the equation of self similar gas motion (cylindrical case) appear in the form of the relations

$$p = p_s(x)\ h(\lambda) \qquad (3.14a)$$

$$\rho = \rho_s(x)\ g(\lambda) \qquad (3.14b)$$

$$v = v_s(x)\ f(\lambda) \qquad (3.14c)$$

$$\lambda = y/y_s \qquad (3.14d)$$

where the index s refers to the condition on the shock-wave. The details of this method are given in Appendix I. The

functions $f(\lambda)$, $g(\lambda)$ and $h(\lambda)$ are also described in Appendix I. The two term outer solution becomes

$$P(X,w) = P_s(X)\ h(\lambda)$$

$$= \left[\frac{2}{\gamma+1}\ A^2 n^2 \frac{1}{X^{n_2}} - \frac{2}{\gamma+1} A^4 n^4 \frac{1}{X^{2n_2}} \right] h(\lambda) \tag{3.15a}$$

$$R(X,w) = R_s g(\lambda)$$

$$= \frac{\gamma+1}{\gamma-1}\ g(\lambda) \tag{3.15b}$$

$$V(X,w) = V_s(X)\ f(\lambda)$$

$$= \left[\frac{2}{\gamma+1}\ An \frac{1}{X^{n_2/2}} - \frac{2}{\gamma+1} A^3 n^3 \frac{1}{X^{3n_2/2}} \right] f(\lambda) \tag{3.15c}$$

$$Y(X,w) = A \lambda X^n \tag{3.15d}$$

Relation between w and λ follows from equation (3.10a)

$$w = \frac{\lambda g(\lambda)}{(\gamma-1)} \left[(\gamma+1) \lambda \frac{A^2}{2} - A^2 f(\lambda) \right] \tag{3.15e}$$

U velocity is determined from equations (3.10e) and (3.12e). Two term outer solution for the velocity turns out to be the

following:

$$U(X,w) = 1 - \frac{2\gamma}{(\gamma+1)^2} A^2 n^2 \left[\frac{f^2(\lambda)}{\gamma} - \frac{h(\lambda)}{g(\lambda)} \right] \frac{1}{X^{n_2}}$$

$$- \frac{1}{2}\left[\left(- \frac{2\gamma}{(\gamma+1)^2} A^2 n^2 \left(\frac{f^2(\lambda)}{\gamma} - \frac{h(\lambda)}{g(\lambda)}\right)\right)^2 \right.$$

$$\left. + \frac{8}{(\gamma+1)^2} A^4 n^4 f^2(\lambda) + \frac{2\gamma}{\gamma-1}\left(- \frac{2(\gamma-1)}{(\gamma+1)^2} A^4 n^4 h(\lambda)\right) \right]$$

$$\frac{1}{X^{2n_2}} \tag{3.15f}$$

Numerical Solution (for the First Outer Equation):

The following differential equations are formed from equations (3.10a–e) for the dependent variables P_1, D_1, U_1, V_1 and Y_1.

$$\frac{dP_1}{dw} = Pf = Pf_1/Pf_2 \tag{3.16a}$$

where

$$Pf_1 = (nY_1{}^2 - V_1Y_1)\frac{D_1 n_1}{\gamma w} + \frac{D_1Y_1V_1n_1}{2w} - V_1/Y_1 \text{ and}$$

$$Pf_2 = \frac{(V_1Y_1 - nY_1{}^2)w^{n_1}}{A\gamma D_1{}^{\gamma-1}} + \frac{D_1Y_1{}^2 n_1}{n_2 w}$$

$$\frac{dD_1}{dw} = \frac{Pfw^{n_1}}{A\gamma D_1^{\gamma-1}} + \frac{n_1 D_1}{\gamma w} \tag{3.16b}$$

$$\frac{dV_1}{dw} = \left(\frac{2Y_1 Pf}{An_2} - V_1\right)\frac{n_1}{2w} \tag{3.16c}$$

$$\frac{dY_1}{dw} = \frac{1}{D_1 Y_1} \tag{3.16d}$$

$$U_1 = -\frac{V_1^2}{2} - \frac{\gamma}{\gamma-1}\frac{P_1}{D_1} \tag{3.16e}$$

Runge-Kutta integration is carried out using the conditions at the shock from equations (3.11a-e).

Inner Expansions:

ψ and x are the independent variables and equations (3.3a-e) describe the inner flow field. The outer solutions are given as functions of λ from the self similar solution. So the inner expansion can-not be arrived at by studying the behaviour of the outer solution as $w \to 0$. However, if we investigate the asymptotic behaviour of the outer equations as w becomes small, the inner expansions can be arrived at.

From 3.10c and 3.12 c we find the asymptotic behaviour of P(X,w) to be

$$P(X,w) = \frac{P_1(w)}{X^{n_2}} + \frac{P_2(w)}{X^{n_2}} \tag{3.17a}$$

where

$$P_1(w) = \frac{V_{1b}}{Y_{1b}} \frac{n_2}{2} w + P_{1b}$$

$$P_2(w) = \frac{V_{2b}}{Y_{2b}} \frac{3n_2}{2} w + P_{2b}$$

where the subscript $(\ )_b$ refers to the flow variables evaluated at the body. So the inner expansion for pressure can be written as follows in terms of inner variables.

$$p(x,\psi) = \frac{p_1(\psi)}{x^{n_2}} + \frac{p_2(\psi)}{x^{2n_2}} + \ldots \tag{3.17b}$$

From similar considerations the expansions for the other variables can be written as follows:

From equations 3.10d and 3.12d we get

$$D(X,w) = D_1(w) + \frac{D_2(w)}{X^{n_2}} \tag{3.17c}$$

where

$$D_1(w) = [P_1(w)]^{1/\gamma} \left(\frac{\gamma+1}{2}\right)^{1/\gamma} \left(\frac{\gamma+1}{\gamma-1}\right) \frac{1}{X^{n_2/\gamma}}$$

$$D_2(w) = \frac{P_2(w)}{P_1(w)} D_1(w)$$

From 3.10a and 3.10e we get

$$Y_1(w) = \frac{2}{[D_1(w)]^{1/2}} \frac{1}{x^n} \qquad (3.17d)$$

$$V_1(w) = nY_1(w) \qquad (3.17e)$$

$$U_1(w) = \frac{\gamma}{\gamma-1} \frac{P_1(w)}{D_1(w)} \qquad (3.17f)$$

Similarly from 3.12a and 3.12e we get

$$Y_2(w) = \frac{V_1(w)U_1(w)}{n} \qquad (3.17g)$$

$$V_2(w) = nY_2(w) \qquad (3.17h)$$

and

$$U_2(w) = V_1(w)V_2(w) \qquad (3.17k)$$

From the above relations the inner expansion is found to be the following:

$$p(x,\psi) = \frac{p_1(\psi)}{x^{n_2}} + \frac{p_2(\psi)}{x^{2n_2}} + \ldots \qquad (3.18a)$$

$$\rho(x,\psi) = \frac{\rho_1(\psi)}{x^{n_2/\gamma}} + \frac{\rho_2(\psi)}{x^{n_2/\gamma + n_2}} + \ldots \qquad (3.18b)$$

$$u(x,\psi) = 1 + \frac{u_1(\psi)}{x^{n_2-n_2/\gamma}} + \frac{u_2(\psi)}{x^{2(n+n_2-n_2/\gamma)}} + \ldots \qquad (3.18c)$$

$$v(x,\psi) = \frac{v_1(\psi)}{x^{\frac{2\gamma-n_2}{2\gamma}}} + \frac{v_2(\psi)}{x^{(1-n_2/2\gamma+n_2-n_2/\gamma)}} + \ldots \qquad (3.18d)$$

$$y(x,\psi) = y_1(\psi)x^{n_2/2\gamma} + y_2(\psi)x^{3n_2/2\gamma - n_2} + \ldots \qquad (3.18e)$$

Substituting these expansions in the equations (3.3a-e) we get the following set of first and second inner equations.

First inner equations:

$$\frac{n_2 y_1}{2\gamma} = v_1 \qquad (3.19a)$$

$$y_1 \frac{dy_1}{d\psi} = \frac{1}{\rho_1} \qquad (3.19b)$$

$$\frac{dp_1}{d\psi} = 0 \qquad (3.19c)$$

$$u_1 + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} = 0 \qquad (3.19d)$$

$$\frac{p_1}{\rho_1^{\gamma}} = \left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma} \frac{2}{\gamma+1} \frac{1}{(1+B\psi^{n_1})} \qquad (3.19e)$$

Second inner equations:

$$y_2\left(\frac{n_2}{2\gamma} - n_2\right) = v_2 \qquad (3.20a)$$

$$y_2\frac{dy_1}{d\psi} + y_1\,\frac{dy_2}{d\psi} = -\,\frac{u_1}{\rho_1} \qquad (3.20b)$$

$$\left.\begin{aligned} \frac{dp_2}{d\psi} &= 0 \qquad (\text{when } n \neq 1/2) \\ \frac{dp_2}{d\psi} &= -\,\frac{v_1}{y_1}\,\left(\frac{n_2 - 2\gamma}{2\gamma}\right) \quad (\text{when } n = 1/2) \end{aligned}\right\} \qquad (3.20c)$$

$$u_2 = 0 \qquad (3.20d)$$

$$p_2 = \frac{\gamma p_1\ \rho_2}{\rho_1} \qquad (3.20e)$$

First Inner Solution:

It is clear from the first inner equations (3.18a-e) that two matching conditions in p and y are required from the outer solution. The following are the conditions written in inner variables.

$$p_1(x,\psi) = \frac{2}{\gamma+1}\,A^2n^2h(\lambda_b)\,\frac{1}{x^{n_2}} \qquad (3.21a)$$

$$y_1(x,\psi) = A\,\lambda_b x^{n_2/2\gamma} \tag{3.21b}$$

When the outer solution is numerically integrated the values of P_1 and Y_1 at $w = 0$ are taken for matching with the inner expansions.

Then the first inner solution is the following:

$$p_1(x,\psi) = \frac{2}{\gamma+1} A^2 n^2 \frac{h(\lambda_b)}{x^{n_2}} \tag{3.22a}$$

$$\rho_1(x,\psi) = \frac{\gamma+1}{\gamma-1}(A^2 n^2 h(\lambda_b))^{1/\gamma}(1+B\psi^{n_1})^{1/\gamma}\frac{1}{x^{n_2/\gamma}} \tag{3.22b}$$

$$u_1(x,\psi) = 1 - \frac{2\gamma}{(\gamma+1)^2}\,\frac{(A^2n^2h(\lambda_b))^{\gamma-1/\gamma}}{(1+B\psi^{n_1})^{1/\gamma}} \Big/ x^{n_2(\gamma-1)/\gamma} \tag{3.22c}$$

$$v_1(x,\psi) = \frac{n_2}{2\gamma}\left\{\left(2.\frac{\gamma-1}{\gamma+1}\right)^{1/2}\left[\frac{1}{A^2n^2h(\lambda_b)}\right]^{1/2\gamma}\frac{(F(\psi))^{1/2}}{x^{1-n_2/2\gamma}} + \frac{A\lambda_b}{x^{1-n_2/2\gamma}}\right\} \tag{3.22d}$$

$$y_1(x,\psi) = \left(2.\frac{\gamma-1}{\gamma+1}F(\psi)\right)^{1/2}\left(\frac{1}{A^2n^2h(\lambda_b)}\right)^{1/2\gamma} x^{n_2/2\gamma} + A\lambda_b x^{n_2/2\gamma} \tag{3.22e}$$

where $F(\psi) = \int \frac{1}{(1+B\psi^{n_1})^{1/\gamma}} d\psi$

Composite Expansion:

The first order solution is given by the construction of the composite expansion written in inner variables as described in Chapter II.

$$\bar{p}_1(x,\psi) = \frac{2}{\gamma+1} A^2 n^2 h(\lambda) \frac{1}{x^{n_2}} \tag{3.23a}$$

$$\bar{\rho}_1(x,\psi) = \frac{\gamma+1}{\gamma-1}\left\{g(\lambda) + [A^2n^2h(\lambda_b)]^{1/\gamma} \frac{[(1+B\psi^{n_1})^{1/\gamma} - (B\psi^{n_1})^{1/\gamma}}{x^{n_2/\gamma}}\right. \tag{3.23b}$$

$$\bar{u}_1(x,\psi) = 1 - \frac{2\gamma}{(\gamma+1)^2} [A^2n^2h(\lambda_b)]^{\gamma-1/\gamma} \left[\frac{1}{(1+B\psi^{n_1})^{1/\gamma}} - \frac{1}{(B\psi^{n_1})^{1/\gamma}}\right.$$

$$- \frac{2\gamma}{(\gamma+1)^2} A^2n^2\left[\frac{f^2(\lambda)}{\gamma} - \frac{h(\lambda)}{g(\lambda)}\right] \frac{1}{x^{n_2}} \tag{3.23c}$$

$$\bar{v}_1(x,\psi) = \frac{2An}{\gamma+1} \frac{f(\lambda)}{x^{n_2/2}} +$$

$$+ \frac{n_2}{2} \left(2.\frac{\gamma-1}{\gamma+1}\right)^{1/2} \left(\frac{1}{A^2n^2h(\lambda_b)}\right)^{1/2} \frac{1}{x^{1-n_2/2\gamma}} [(F(\psi))^{1/2} - F_1(\psi)] \tag{3.24d}$$

$$\bar{y}_1(x,\psi) = A\lambda x^{n} + x^{n_2/2\gamma}\left[\left(2\frac{\gamma-1}{\gamma+1}\right)^{1/2}\left(\frac{1}{A^2n^2h(\lambda_b)}\right)^{1/2\gamma}\right.$$

$$\left.((F(\psi))^{1/2} - F_1(\psi))\right] \tag{3.23e}$$

where

$$F_1(\psi) = \underset{\psi\to\infty}{\mathrm{Lt}} \int \frac{d\psi}{(1+B\psi^{n_1})^{1/\gamma}}$$

Second Inner Solution:

Here again the matching conditions are

$$p_2(x,\psi) = -\frac{2}{\gamma+1}A^4n^4h(\lambda_b)\frac{1}{x^{2n_2}} \tag{3.24a}$$

$$y_2(x,\psi) = 0 \tag{3.24b}$$

From equation (3.20b)

$$y_2 = \frac{1}{y_1}\int F_2(\psi)d\psi \tag{3.25a}$$

where

$$F_2(\psi) = -\frac{\left[1-\frac{2\gamma}{(\gamma+1)^2}\frac{A^2n^2h(\lambda_b)}{(1+B\psi^{n_1})^{1/\gamma}}\right]^{\gamma-1/\gamma}}{\frac{\gamma+1}{\gamma-1}(A^2n^2h(\lambda_b))^{1/\gamma}(1+B\psi^{n_1})^{1/\gamma}}$$

$$p_2 = -\frac{2}{\gamma+1}A^4n^4h(\lambda_b) \quad (\text{when } n\neq 1/2) \tag{3.25b}$$

$$p_2 = -\frac{n_2}{(2\gamma)^2}(n_2-2\gamma)\psi - \frac{2}{\gamma+1}A^4n^4h(\lambda_b) \qquad (3.25b_2)$$

(when n = 1/2)

$$v_2 = \left(\frac{n_2}{2\gamma} - n_2\right)\frac{1}{y_1}\int F_2(\psi)d\psi \qquad (3.25c)$$

$$\rho_2 = \frac{p_2}{\gamma}\frac{\rho_1}{p_1} \qquad (3.25d)$$

$$u_2 = 0 \qquad (3.25e)$$

The second inner solution can be written directly from (3.25a-e)

$$p_2(x,\psi) = p_2/x^{2n_2} \qquad (3.26a)$$

$$\rho_2(x,\psi) = \rho_2/x^{n_2/\gamma + n_2} \qquad (3.26b)$$

$$u_2(x,\psi) = 0 \qquad (3.26c)$$

$$v_2(x,\psi) = v_2/x^{(2\gamma-n_2)/2\gamma + (n_2-n_2/\gamma)} \qquad (3.26d)$$

$$y_2(x,\psi) = y_2\, x^{3n_2/\gamma - n_2} \qquad (3.26e)$$

The composite expansion for the second term can be found out in the same manner as before

$$\bar{p}_2(x,\psi) = -\frac{2}{\gamma+1} A^4 n^4 h(\lambda)/x^{2n_2} \quad \text{(when } n \neq 1/2) \qquad (3.27a_1)$$

$$\bar{p}_2(x,\psi) = -\left[\frac{2}{\gamma+1} A^4 n^4 h(\lambda) + \frac{n_2}{(2\gamma)^2}(n_2-2\gamma)\psi\right]/x^{2n_2}$$

$$\text{(when } n = 1/2) \qquad (3.27a_2)$$

$$\bar{\rho}_2(x,\psi) = \rho_2(x,\psi) \qquad (3.27b)$$

$$\bar{u}_2(x,\psi) = -\frac{1}{2}\left[\left\{-\frac{2\gamma}{(\gamma+1)^2} A^2 n^2\left(\frac{f^2(\lambda)}{\gamma} - \frac{h(\lambda)}{g(\lambda)}\right)\right\}^2\right.$$

$$\left. + \frac{8}{\gamma+1} A^4 n^4 f^2(\lambda) + \frac{2\gamma}{\gamma-1}\left(-\frac{2(\gamma-1)}{(\gamma+1)^2} A^4 n^4 h(\lambda)\right)\right]/x^{2n_2}$$

$$(3.27c)$$

$$\bar{v}_2(x,\psi) = -\frac{2}{\gamma+1} A^3 n^3 \frac{1}{x^{3n_2/2}} f(\lambda) + v_2(x,\psi)$$

$$-\left(\frac{n_2}{2\gamma} - n_2\right) F_3(\psi)/x^{(2\gamma-n_2)/2\gamma + (n_2-n_2/\gamma)}$$

$$(3.27d)$$

$$\bar{y}_2(x,\psi) = y_2(x,\psi) - F_3(\psi)/x^{(2\gamma-n_2)/2\gamma + (n_2-n_2/\gamma)}$$

$$(3.27e)$$

where $F_3(\psi) = \underset{\psi \to \infty}{\mathrm{Lt}} \frac{1}{y_1} \int F_2(\psi)\,d\psi$

The complete second order solution follows from equations (3.23a-c) and (3.27a-e)

$$\bar{p}(x,\psi) = \bar{p}_1(x,\psi) + \bar{p}_2(x,\psi) \qquad (3.28a)$$

$$\bar{\rho}(x,\psi) = \bar{\rho}_1(x,\psi) + \bar{\rho}_2(x,\psi) \qquad (3.28b)$$

$$\bar{u}(x,\psi) = \bar{u}_1(x,\psi) + \bar{u}_2(x,\psi) \qquad (3.28c)$$

$$\bar{v}(x,\psi) = \bar{v}_1(x,\psi) + \bar{v}_2(x,\psi) \qquad (3.28d)$$

$$\bar{y}(x,\psi) = \bar{y}_1(x,\psi) + \bar{y}_2(x,\psi) \qquad (3.28e)$$

The results of the second order solution are given in Figures (8-13).

CHAPTER IV

PLANE POWER LAW SHOCK

The shock is given by

$$y = Ax^n \tag{4.1}$$

From Equation (2.5) the form of the stream function is given by

$$\psi = Ax^n \tag{4.2}$$

The basic equations from (3.3a-e) for the plane case are as follows:

$$\frac{\partial y}{\partial x} = \frac{v}{u} \tag{4.3a}$$

$$\frac{\partial y}{\partial \psi} = \frac{1}{\rho u} \tag{4.3b}$$

$$\frac{\partial v}{\partial x} + \frac{\partial p}{\partial \psi} = 0 \tag{4.3c}$$

$$u^2 + v^2 + \frac{2\gamma}{\gamma-1}\frac{p}{\rho} = 1 \tag{4.4d}$$

$$p/\rho^\gamma = \left(\frac{\gamma-1}{\gamma+1}\right)^\gamma \frac{2}{\gamma+1} \sin^2\sigma \tag{4.4e}$$

Sin σ and cos σ are found in terms of ψ as follows

$$\sin^2\sigma = \frac{B_1^2 \cdot \psi^2}{\psi^{2/n} + B_1^2\, \psi^{2n_{11}}} =$$

$$\cos^2\sigma = \frac{\psi^{1/n}}{\sqrt{\psi^{2/n} + B_1^2\, \psi^2}}$$

where

$$B_1^2 = n^2\, A^{2/n} \quad \text{and} \quad n_{11} = 1-n$$

Boundary conditions follow from (2.4)

$$p = \frac{2}{\gamma+1}\, \frac{B_1^2\, \psi^2}{\psi^{2/n} + B_1^2\, \psi^2} \tag{4.5a}$$

$$\rho = \frac{\gamma+1}{\gamma-1} \tag{4.5b}$$

$$u = 1 - \frac{2}{\gamma+1}\, \frac{B_1^2\, \psi^2}{\psi^{2/n} + B_1^2\, \psi^2} \tag{4.5c}$$

$$v = \frac{2}{\gamma+1}\, \frac{B_1\, \psi\, \psi^{1/n}}{\psi^{2/n} + B_1^2\, \psi^2} \tag{4.5d}$$

$$y = \psi$$

Outer Expansions

X and w are the outer independent variables where

$$X = x \qquad \text{and} \tag{4.6a}$$

$$w = \psi/x^n \tag{4.6b}$$

The transformed equations of (3.3a-e) with the corresponding flow variables replaced by P, D, V, U and Y, are the following:

$$\frac{\partial Y}{\partial X} = \frac{V}{U} + \frac{A\,w\,n}{D\,U}\,X^{n-1} \tag{4.7a}$$

$$\frac{\partial Y}{\partial w} = \frac{1}{DU}\,AX^n \tag{4.7b}$$

$$AX^n\,\frac{\partial V}{\partial X} - AX^{n-1}\,w\,n\,\frac{\partial V}{\partial w} + \frac{\partial P}{\partial w} = 0 \tag{4.7c}$$

$$U^2 + V^2 + \frac{2\gamma}{\gamma-1}\,\frac{P}{D} = 1 \tag{4.7d}$$

$$P/D^{\gamma} = \left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma}\frac{2}{\gamma+1}\left(\frac{B_1^2\,w}{A^{2/n}\,X^{2(1-n)}}\right)^{2(1-\frac{1}{n})}$$

$$\left[\,1 - \frac{B_1^2}{A^{2/n}\,w^{2(1-n)/n}\,X^{2(1-n)}} + \dots\,\right] \tag{4.7e}$$

The shock conditions are transformed into the following.

$$P = \frac{2}{\gamma+1} \frac{B_1^2}{A^{2/n} w^{2(1-n)/n} X^{2(1-n)}} \left[1 - \frac{B_1^2}{A^{4/n} w^{2(1-n)/n} X^{2(1-n)}} + \cdots \right] \tag{4.8a}$$

$$D = \frac{\gamma+1}{\gamma-1} \tag{4.8b}$$

$$U = 1 - \frac{2}{\gamma+1} \frac{B_1^2}{A^{2/n} w^{2(1-n)/n} X^{2(1-n)}} \left[1 - \frac{B_1^2}{A^{4/n} w^{2(1-n)/n} X^{2(1-n)}} + \cdots \right] \tag{4.8c}$$

$$V = \frac{2}{\gamma+1} B_1 \left[\frac{1}{A^{(\frac{1}{n}-1)} w^{\frac{1-n}{n}} X^{(1-n)}} \right] \left[1 - \frac{B_1^2}{w^{2(1-n)/n} X^{2(1-n)}} + \cdots \right] \tag{4.8d}$$

$$Y = A w X^n \tag{4.8e}$$

evaluated at $w = A$.

Outer Expansions:

The asymptotic expansions for the outer flow quantities can be directly written from the shock conditions.

$$P(X,w) = \frac{P_1(w)}{X^{2n_{11}}} + \frac{P_2}{X^{4n_{11}}} + \cdots \tag{4.9a}$$

$$D(X,w) = D_1(w) + \frac{D_2(w)}{X^{2n_{11}}} + \cdots \tag{4.9b}$$

$$U(X,w) = 1 + \frac{U_1(w)}{X^{2n_{11}}} + \frac{U_2(w)}{X^{4n_{11}}} + \cdots \tag{4.9c}$$

$$V(X,w) = \frac{V_1(w)}{X^{n_{11}}} + \frac{V_2(w)}{X^{3n_{11}}} + \cdots \tag{4.9d}$$

$$Y(X,w) = X^{1-n_{11}} \left[Y_1(w) + \frac{Y_2(w)}{X^{2n_{11}}} + \cdots \right] \tag{4.9e}$$

Substituting the above expansions into the equations (4.7a-e) we get the outer equations. Since the procedure is same as for the axi-symmetric case only the first order solution is given in this chapter.

First Outer Equation:

$$nY_1 = V_1 + \frac{A\,w\,n}{D_1} \tag{4.10a}$$

$$\frac{dY_1}{dw} = \frac{A}{D_1} \tag{4.10b}$$

$$- AV_1 n_{11} - Awn \frac{dV_1}{dw} + \frac{dP_1}{dw} = 0 \tag{4.10c}$$

$$2U_1 + V_1^{\,2} + \frac{2\gamma}{\gamma-1} \frac{P_1}{D_1} = 0 \tag{4.10d}$$

$$\frac{P_1}{D_1^{\gamma}} = \left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma} \frac{2}{\gamma+1} \frac{B_1^2 \, w^{2(n-1)/n}}{A^{2/n}} \tag{4.10e}$$

Boundary conditions

$$P = \frac{2}{\gamma+1} A^2 n^2 \tag{4.11a}$$

$$D = \frac{\gamma+1}{\gamma-1} \tag{4.11b}$$

$$U = -\frac{2}{\gamma+1} A^2 n^2 \tag{4.11c}$$

$$V = \frac{2}{\gamma+1} An \tag{4.11d}$$

$$Y = A \tag{4.11e}$$

Here again the outer solution is same as the self similar solution (plane case), the details of which are given in Appendix I. The first outer solution is the following.

$$P_1(X,w) = \frac{2}{\gamma+1} A^2 n^2 \; h(\lambda) \frac{1}{X^{2n_{11}}} \tag{4.12a}$$

$$D_1(X,w) = \frac{\gamma+1}{\gamma-1} \; g(\lambda) \tag{4.12b}$$

$$U_1(X,w) = 1 - \frac{2}{(\gamma+1)^2} A^2 n^2 \left[f^2(\lambda) + \frac{h(\lambda)}{g(\lambda)} \gamma \right] \frac{1}{X^{2n_{11}}} \tag{4.12c}$$

$$V_1\ (X,w) = \frac{2}{\gamma+1} A\, n\, f(\lambda)\, \frac{1}{X^{n_{11}}} \tag{4.12d}$$

$$Y_1\ (X,w) = A \lambda X^n \tag{4.12e}$$

Relation between w and λ follows from the equation (4.10a)

$$w = \frac{g(\lambda)}{\gamma-1} \left[(\gamma+1)\lambda - 2f(\lambda) \right]$$

Inner Expansion:

ψ and x are the independent variables and equations (4.3a-e) describe the inner flow field. For finding out the inner expansion, the same procedure as for the axi-symmetric case is adopted and the following expansions are arrived at.

$$p\ (x,\psi) = p_1(\psi)/x^{2n_{11}} \tag{4.13a}$$

$$\rho\,(x,\psi) = \frac{\rho_1(\psi)}{x^{2n_{11}/\gamma}} \tag{4.13b}$$

$$u\ (x,\psi) = 1 + \frac{u_1\ (\psi)}{x^{2n_{11}(\gamma-1)/\gamma}} + \cdots \tag{4.13c}$$

$$v\ (x,\psi) = \frac{v_1\ (\psi)}{x^{(\gamma-2n_{11})/\gamma}} + \cdots \tag{4.13d}$$

$$y = y_1\ (\psi)\ x^{2n_{11}/\gamma} + \cdots \tag{4.13e}$$

Substituting these expansions in the equations (4.3a–e) we get the following inner equation

$$2n_{11} y_1 = \gamma v_1 \tag{4.14a}$$

$$\frac{dy_1}{d\psi} = \frac{1}{\rho_1} \tag{4.14b}$$

$$\frac{dp_1}{d\psi} = 0 \tag{4.14c}$$

$$u_1 + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} = 0 \tag{4.14d}$$

$$\frac{p_1}{\rho_1^{\gamma}} = \left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma} \frac{2}{\gamma+1} \frac{B_1^2 \psi^2}{\psi^{2/n} + B_1 \psi^2} \tag{4.14e}$$

The two matching conditions required to solve the inner equations are the following

$$p_1 (x,\psi) = \frac{2}{\gamma+1} A^2 n^2 h (\lambda_b) \frac{1}{x^{2n_{11}}}$$

$$y_1 (x,\psi) = A \lambda_b x^{2n_{11}/\gamma}$$

From equation 4.14c

$$p_1 = \frac{2}{\gamma+1} A^2 n^2 h (\lambda_b)$$

$$\rho_1 = F(\psi)$$

where

$$F(\psi) = \frac{\gamma+1}{\gamma-1}\left[\frac{2}{\gamma+1} A^2 n^2 h(\lambda_b)\right]^{1/\gamma}\left[\frac{\gamma+1}{2B_1^2}\right]^{1/\gamma}\left[\frac{\psi^{2/n} + B_1^2\psi^2}{\psi^2}\right]^{1/\gamma}$$

$$y_1 = \int F(\psi)\, d\psi + A\lambda_b$$

$$v_1 = \frac{1}{2n_{11}}\left[\int F(\psi)\, d\psi + A\lambda_b\right]$$

$$u_1 = \frac{-2\gamma}{(\gamma+1)^2}\ \frac{A^2 n^2 h(\lambda_b)}{F(\psi)}$$

Composite Expansion:

$$\bar{p}_1(x, \psi) = \frac{2}{\gamma+1} A^2 n^2 h(\lambda) \frac{1}{x^{2n_{11}}}$$

$$\bar{\rho}_1(x, \psi) = \frac{\gamma+1}{\gamma-1} g(\lambda) + [F(\psi) - F_1(\psi)] \frac{1}{x^{2n_{11}/\gamma}}$$

$$\bar{u}_1(x, \psi) = 1 - \frac{2}{\gamma+1} A^2 n^2 \left[f^2(\lambda) + \frac{h(\lambda)}{g(\lambda)}\gamma\right] \frac{1}{x^{2n_{11}}}$$

$$- \frac{2\gamma}{(\gamma+1)^2} A^2 n^2 h(\lambda_b)\left[\frac{1}{F(\psi)} - \frac{1}{F_1(\psi)}\right]$$

$$\bar{v}_1 \ (x, \psi) = \frac{2}{\gamma+1} \Lambda \, n \, f \ (\lambda) \ \frac{1}{x^{n_1}} + \frac{1}{2n_{11}} \left[\int F \ (\psi) \ d\psi - \underset{\psi \to \infty}{\mathrm{Lt}} \int F(\psi) d\psi \right] \frac{1}{x^{(\gamma - 2n_{11})/\gamma}}$$

$$\bar{y}_1 \ (x, \psi) = \Lambda \lambda x^n + \left[\int F \ (\psi) d\psi - \underset{\psi \to \infty}{\mathrm{Lt}} \int F \ (\psi) \ d\psi \right] x^{2n_{11}/\gamma}$$

where $F_1 \ (\psi) = \underset{\psi \to \infty}{\mathrm{Lt}} \ F \ (\psi)$

The body for the case $n = 2/3$ is given in Fig. (14). The flow profiles are also given in Figs. (15-16).

CHAPTER 5

DISCUSSIONS AND CONCLUSIONS

The results of the first order axisymmetric body for n in the range '$2/3+j < n < 1$' are given in Figures (2-7). For the two cases, when $n = 1/2$ and $n = 2/3$, second order results are given in Figures (2 - 3). Also the results of the first and second order solutions for the flow properties are given in Figures (8-13). In the case of the plane flow the results of the first order body and the flow properties are given in Figures (14-16), for $n = 2/3$. Ratio of the body to the shock thickness for the axisymmetric case is given in Figure (17).

In Figure (2), the first order results are the same as Yakura's results for $n = 1/2$. The difference, in the body shape by Sychev and Yakura, is due to the velocity-defect effect, which is included in Sychev's calculations but is not included in the first order theory of Yakura. The numerical solution obtained by Sychev is of a higher order than the first because he has used the two-term approximation for the velocity u and the one-term approximation for the density. It is in close agreement with the present second order results. As n increases the body approaches the shock. Figure (17) shows that the entropy layer is of negligible importance when $n > 0.65$. In

Figure (7) for $n = 0.99$, the body is almost straight making an angle of 36.5° which is very nearly the same as the flow deflection angle for a shock angle of 45°. This body coincides with Yakura's body (producing a hyperboloidal shock) for large distances.

The first order pressure gradient is zero and the second order pressure gradient exists only when the power-law exponent, n, is 1/2. In the outer region the first order and the second order densities coincide. Figures (10 and 11) show that the u velocity decreases as we go towards the body and the gradients are more predominent in the inner region. Figures (9 and 12) show the gradient of the entropy function. As we go away from the leading edge of the body the gradient occurs only near the body. The entropy layer thickness decreases away from the leading edge but it exists even for large distances. The second order results are not very significant though the correct the body and the flow properties in the right direction.

The entropy layer and the Prandtl's boundary layer have common features like the displacement thickness and zero transverse pressure gradients. The influence of the entropy layer varies with the body or shock geometry and is more predominent for $n < n^*$. Hornung[10] has discussed the range of n for the attached layer, the free layer and the blast wave region. The flow in the entropy layer has gone

through a strong shock and does not satisfy the hypersonic slender body requirements. As a consequence the entropy layer flow does not exhibit similitude (6). But the self-similar solutions become significant for larger distances from the nose of the body, and also when n is in the range, $n^* < n < 1$.

APPENDIX I

SIMILAR POWER LAW SOLUTIONS

In general, the solution of the partial differential equations governing a gas dynamic situation, can only be found by complicated numerical methods. But for certain gas motions, it turns out that these equations can be reduced to ordinary differential equations and one can obtain exact solutions in closed form or approximate solutions through the use of comparatively simple numerical integration methods. Such motions are called self-similar.

The solution to the small disturbance equations of the steady two dimensional or axisymmetric hypersonic flows come under the class of similar solutions. The equations governing hypersonic flow over slender two dimensional or axisymmetric body are similar to the unsteady hypersonic flow equations in one space dimension, the solution of which is known (4). This and the fact that the hypersonic motion of a slender body through a gas produces only a transverse displacement of the gas (hypersonic equivalence principle) helps in assuming that the solution at one section (at a point on the body perpendicular to the flow direction) is similar to the one at any other section on the axis.

One-dimensional Unsteady Equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial y}(\rho v) + j y^{-1} \rho v = 0 \qquad \text{(A.1a)}$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \qquad \text{(A.1b)}$$

$$\frac{\partial}{\partial t}(p/\rho^{\gamma}) + v \frac{\partial}{\partial y}(p/\rho^{\gamma}) = 0 \qquad \text{(A.1c)}$$

$j = 0$ Plane flow

$j = 1$ Axisymmetric flow

where p, ρ and v are the pressures, density and velocity in the y direction respectively. The equations A.1 can be transformed into the hypersonic small disturbance equation by a simple transformation $x = U_\infty t$ where U_∞ is the free stream velocity. In general, Hayes transformation enables one to transform the approximate equation representing a steady n-dimensional flow into exact equation representing an unsteady n-1-dimensional flow. Thus all the results of the unsteady gasdynamics can be used to investigate the corresponding approximate steady flow. The time axis transforms into the x axis. This principle is called Hayes equivalence principle or the 'Law of Plane Sections'.

Solution of the Exact Equations:

Dimensionless variables V, R, and P are defined to replace the dependent variables

$$v = yt^{-1}V \tag{A.2a}$$

$$\rho = Ay^{-w}R \tag{A.2b}$$

$$p = Ar^{2-w}t^{-2}P \tag{A.2c}$$

where A is a constant, the dimensions of which also contain the mass since the dimensions of the quantities ρ and p also contain the mass; w is a dimensionless parameter.

An additional parameter B is introduced so that its dimensions do not contain the mass, and number of independent variables is reduced to one by defining a new independent variable λ,

$$\lambda = Byt^{-k} \tag{A.3}$$

where k is a dimensionless exponent. The dimensionless variable λ is constant on a path for which y is proportional to t^k or in an equivalent steady hypersonic flow to x^k. Substituting A.2 and A.3 in A.1, we get the following set of ordinary differential equations:

$$\lambda \left[(V-k)V' + \frac{P'}{R} \right] = -V(V-1) - (2-w)\frac{P}{R} \tag{A.4a}$$

$$\lambda\left[V' + (V-k)\frac{R'}{R} \right] = - (1+j-w)V \qquad (A.4b)$$

$$\left[\lambda(V-k) \right] \left[\frac{P'}{P} - \gamma \frac{R'}{R} \right] = -2(V-1) - w(\gamma-1)V$$

$$(A.4c)$$

Here the primes indicate differentiation with respect to λ. Body or a shock-wave in the flow field must lie on a line of constant λ.

A new independent variable z is introduced to replace the variable P.

$$z = \gamma\frac{P}{R} \qquad (A.5)$$

A single differential equation is obtained from A.4 in which P, R and [illegible] do not appear.

$$\frac{dz}{dV} = \frac{z\left\{[2(V-1)+j(\gamma-1)V](V-k)^2+(\gamma-1)(1-k)V(V-k)-[2(V-1)+K(\gamma-1)z]\right\}}{(V-k)\left\{V(V-1)(V-k)-[(1+j)V-K]z\right\}}$$

$$(A.6)$$

where $K = \left[2(1-k) + kw \right]/\gamma$

From the same equation A.5 following relations may be obtained.

$$\frac{d \log \lambda}{dV} = \frac{z-(V-k)^2}{V(V-1)(V-k)-[(1+j)V-K]z} \qquad (A.7a)$$

$$\frac{d\log(V-k)R}{d\log\lambda} = -\frac{(1+j-w)V}{(V-k)} \tag{A.7b}$$

Once a solution is obtained from A.6 for $z(V)$, λ and R may be obtained by quadratures over V.

Boundary Conditions:

A similarity line is defined as the line in (y,t) space for which λ is constant and y is proportional to t^k. The velocity of a point on this line is kyt^{-1}. From A.2a the velocity of a fluid particle relative to a similarity line is $(V-k)yt^{-1}$. Thus $V=k$ is the boundary condition on a physical boundary which follows a similarity line.

In the undisturbed region of flow the pressure must be constant in both time and space. From A.2.c either $k=2/(2-w)$ with P proportional to $\lambda^{(-2+w)}$ or $P = 0$. Except when $k=2/(2-w)$ the pressure in the undisturbed region in front of a shock must be negligibly small. Also density in the undisturbed region must be independent of t. From A.2b it can be deduced that R is a constant and the density is proportional to a power $-w$ of y. In our problem the density is a constant in front of the shock and so w is taken to be zero. The shock conditions give us the boundary values for the variables immediately behind the shock.

$$V_s = \frac{2k}{\gamma+1} \tag{A.8a}$$

$$R_s = \frac{\gamma+1}{\gamma-1} \tag{A.8b}$$

$$P_s = \frac{2k^2}{\gamma+1} \tag{A.8c}$$

$$z_s = \frac{2\gamma(\gamma-1)k^2}{(\gamma+1)^2} \tag{A.8d}$$

In the solutions of interest to us which contain the shocks, the location of the body is unknown in advance. The fact that this boundary is a floating one requires that we impose an additional condition to determine the solution,

$$V_b = k$$

where suffix b refers to the physical boundary.

Constant Energy Solution:

Sedov[4] obtained unique integrals of equation A.6 for the case of constant energy by considering conservation of energy across two similarity lines λ_1 and λ_2.

$$z \equiv (\gamma-1)V^2(k-V)/2(V-k/\gamma) \tag{A.9}$$

Although the integral A.9 is obtained from considerations entirely from those, we obtained equation A.6, yet by

actual substitution we find that A.9 satisfies A.6 exactly. For this solution k is given by

$$k = 1/2+j-w \tag{A.10a}$$

when $w = 0$ constant density case,

$$k = 1/2+j \tag{A.10b}$$

The solution A.9 satisfies the shock boundary condition A.8d. The quantity z is infinite at $V=k/\gamma$. By actual computation we find that as $\lambda \to 0$, $V \to k/\gamma$ and $z \to \infty$. So we may set the inner boundary condition

$$\lambda_b = 0 \ , \quad V_b = k/\gamma \tag{A.11}$$

This shows that at the core when $\lambda_b=0$ or $y=0$ the density $\rho \to 0$ and the temperature $T \to \infty$ while the pressure remains finite. The quadratures of A.7 together with the basic solution A.6 yield the results for a given value of time or x as the case may be

$$\frac{y}{y_s} = \lambda$$

$$\frac{v}{v_s} = \frac{\gamma+1}{2k} \lambda V = f$$

$$\frac{\rho}{\rho_s} = \frac{\gamma-1}{\gamma+1} \ R = g$$

$$\frac{p}{p_s} = \frac{\gamma + 1}{2k^2} \lambda^2 P = h$$

The functions given in (4) f, g, h are tabulated for the case of violent explosion when j = 0 or 1 in the following pages.

TABLE 1

$j = 0 \qquad \gamma = 1.4 \qquad k = 2/3$

λ	f	g	h
1.0000	1.0000	1.0000	1.0000
0.9797	0.9699	0.8625	0.9162
0.9420	0.9156	0.6659	0.7915
0.9013	0.8599	0.5160	0.6923
0.8565	0.8017	0.3982	0.6120
0.8050	0.7390	0.3019	0.5457
0.7419	0.6678	0.2200	0.4904
0.7029	0.6263	0.1823	0.4661
0.6553	0.5780	0.1453	0.4437
0.5925	0.5172	0.1074	0.4229
0.5396	0.4682	0.0826	0.4116
0.4912	0.4244	0.0641	0.4038
0.4589	0.3957	0.0536	0.4001
0.4161	0.3580	0.0415	0.3964
0.3480	0.2988	0.0263	0.3929
0.2810	0.2410	0.0153	0.3911
0.2320	0.1989	0.0095	0.3905
0.1680	0.1441	0.0042	0.3901
0.1040	0.0891	0.0013	0.3900
0.0000	0.0000	0.0000	0.3900

TABLE 2

j = 1 γ = 1.4 k = 1/2

λ	f	g	h
1.0000	1.0000	1.0000	1.0000
0.9988	0.9996	0.9973	0.9985
0.9802	0.9645	0.7653	0.8659
0.9649	0.9374	0.6285	0.7832
0.9476	0.9097	0.5164	0.7124
0.9295	0.8812	0.4234	0.6514
0.9096	0.8514	0.3451	0.5983
0.8725	0.7998	0.2427	0.5266
0.8442	0.7638	0.1892	0.4884
0.8094	0.7226	0.1414	0.4545
0.7629	0.6720	0.0975	0.4242
0.7242	0.6327	0.0718	0.4074
0.6894	0.5989	0.0595	0.3969
0.6390	0.5521	0.0362	0.3867
0.5745	0.4943	0.0208	0.3794
0.5180	0.4448	0.0123	0.3760
0.4748	0.4073	0.0079	0.3746
0.4222	0.3621	0.0044	0.3737
0.3654	0.3133	0.0021	0.3733
0.3000	0.2571	0.0008	0.3730
0.2500	0.2143	0.0003	0.3729
0.1500	0.1286	0.0000	0.3729
0.0000	0.0000	0.0000	0.3729

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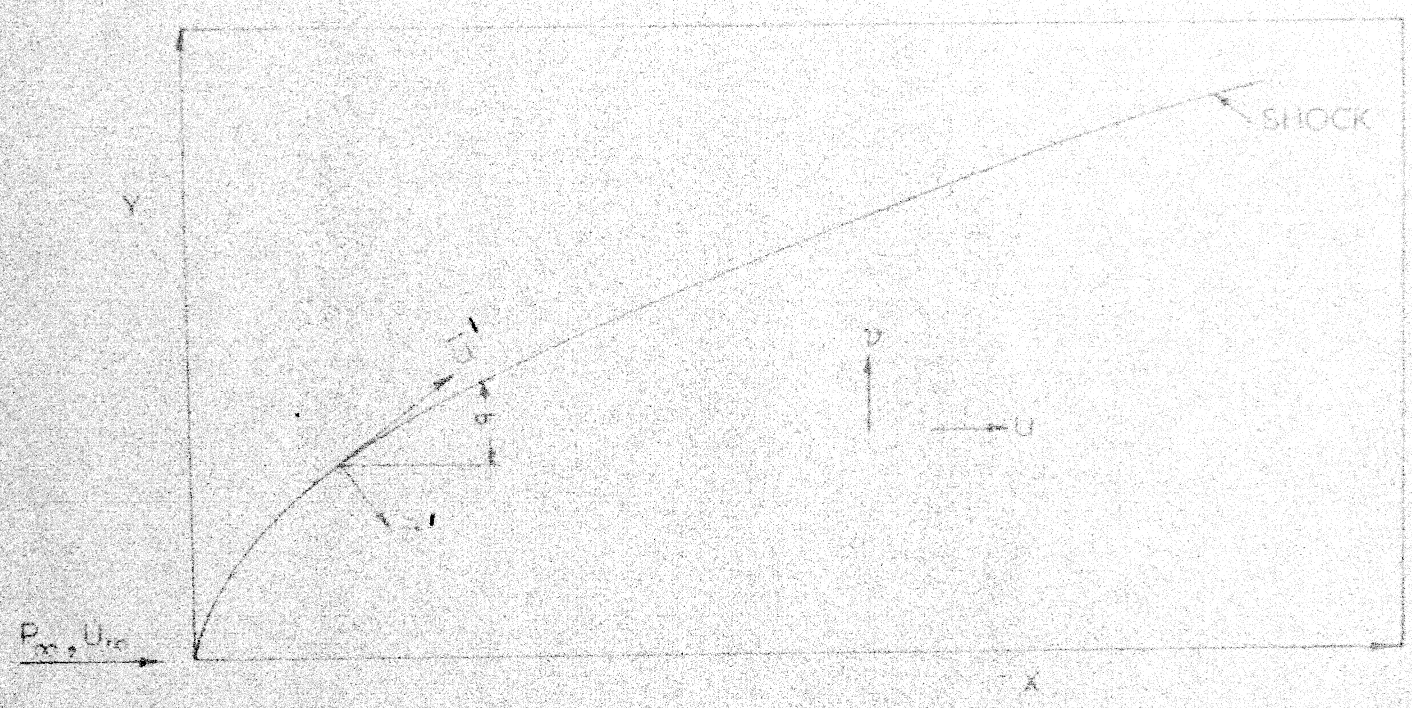

FIG 1. POWER LAW SHOCK

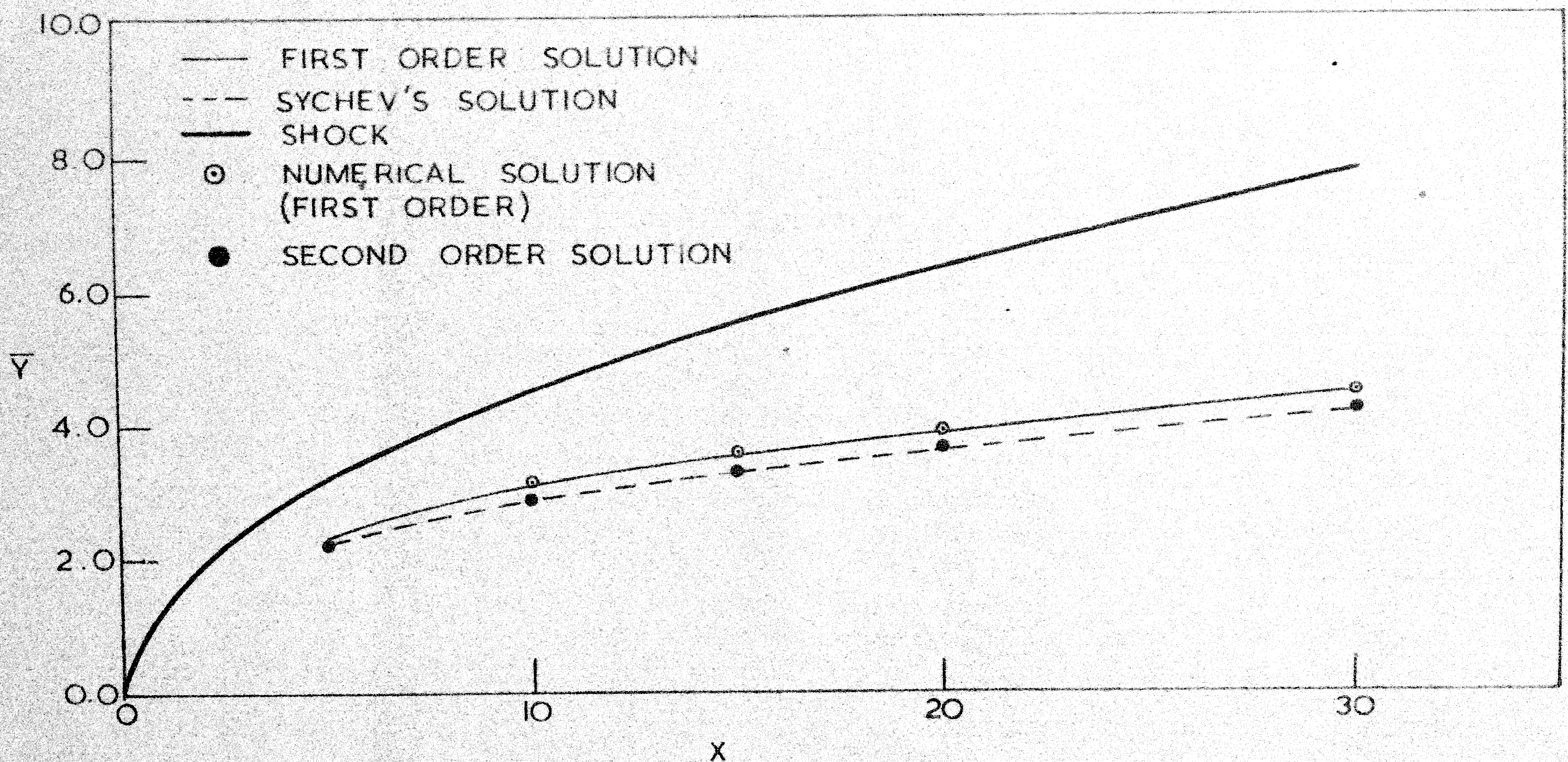

FIG. 2 – BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = \sqrt{2} . X^{1/2}$ WITH $M = \infty$ AND $\gamma = 1.4$

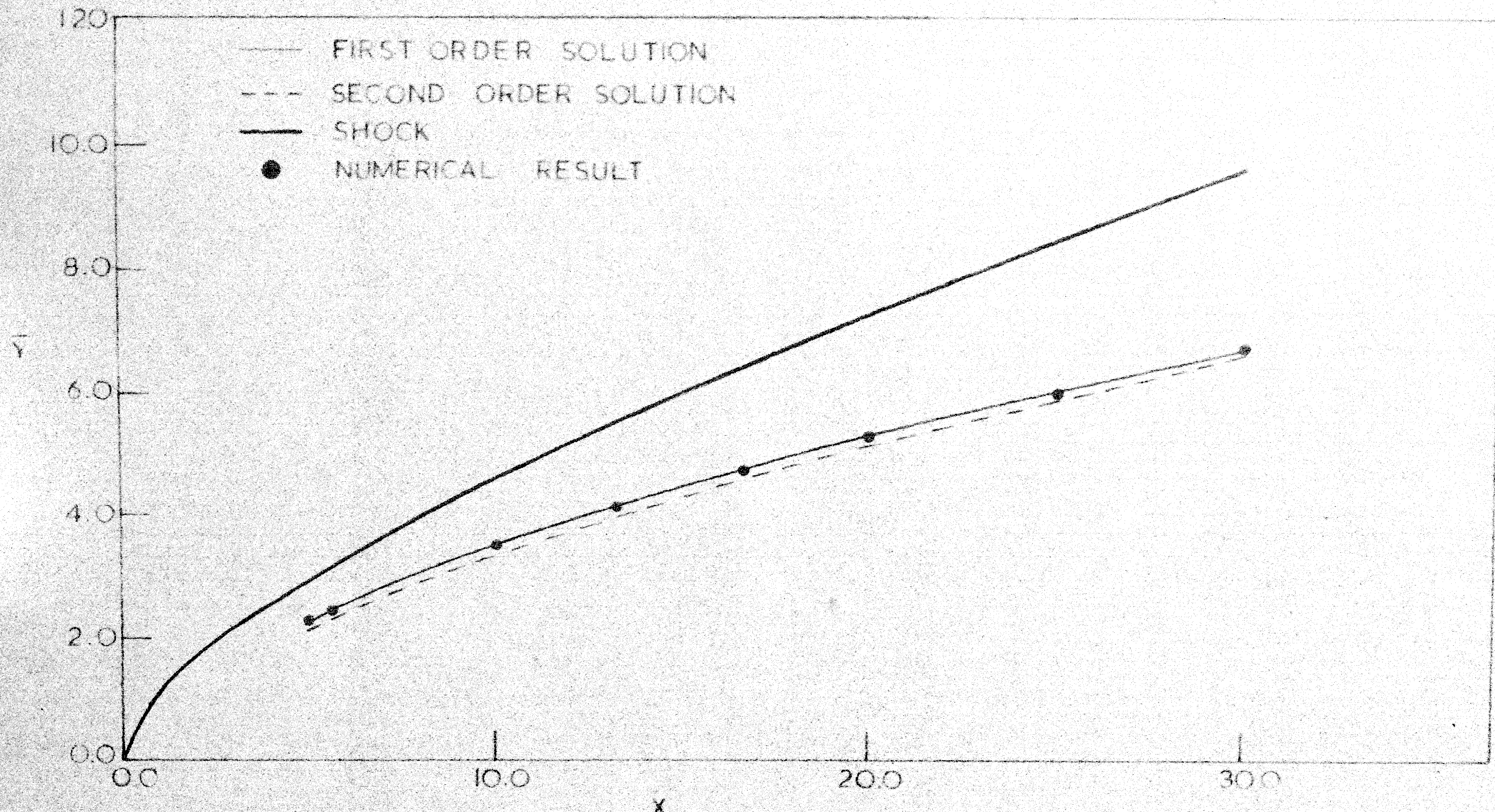

FIG. 3 – BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$ WITH $M = \infty$ AND $\gamma = 1.4$

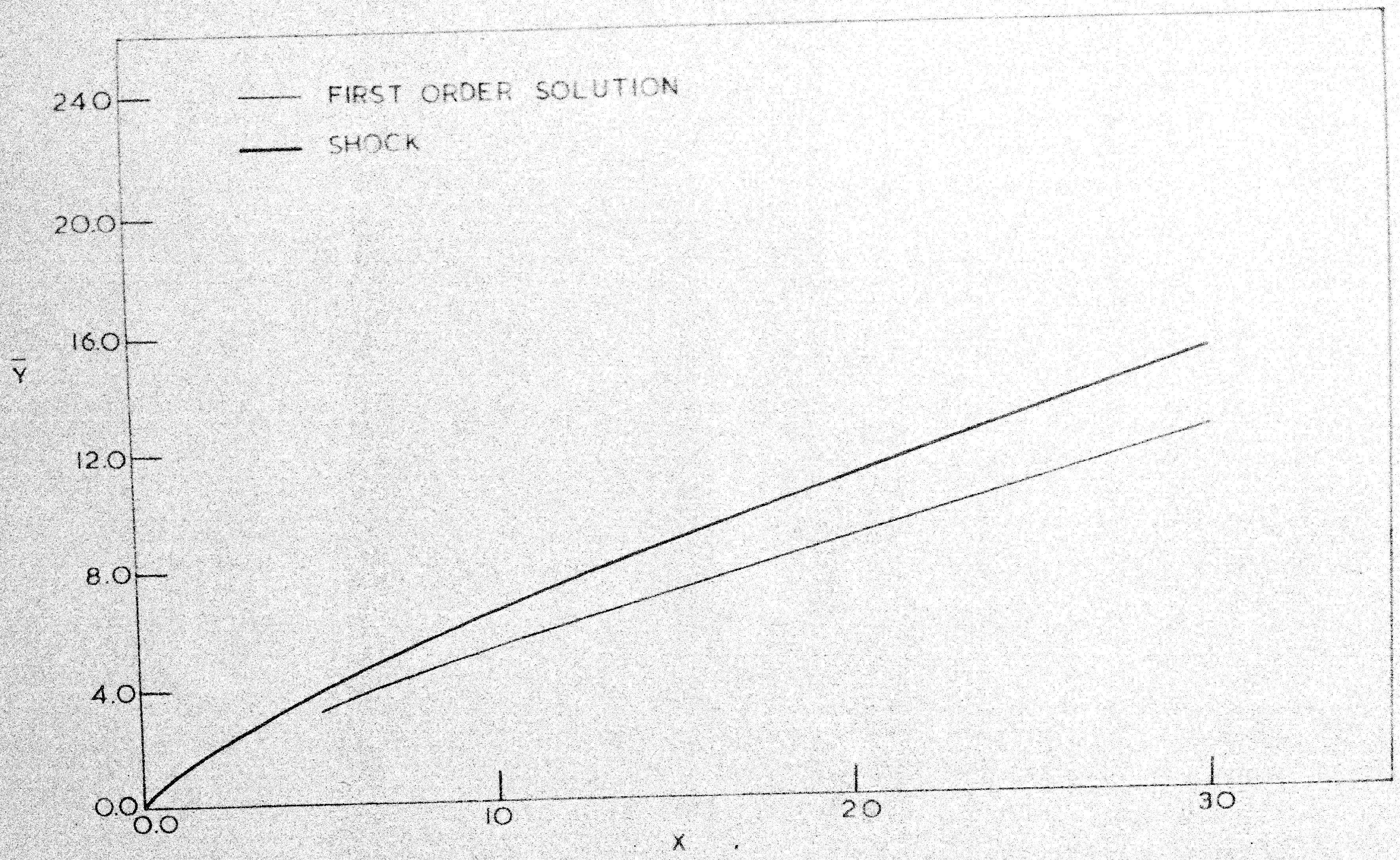

FIG. 5 _ BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{4/5}$ WITH $M = \infty$ AND $\gamma = 1.4$

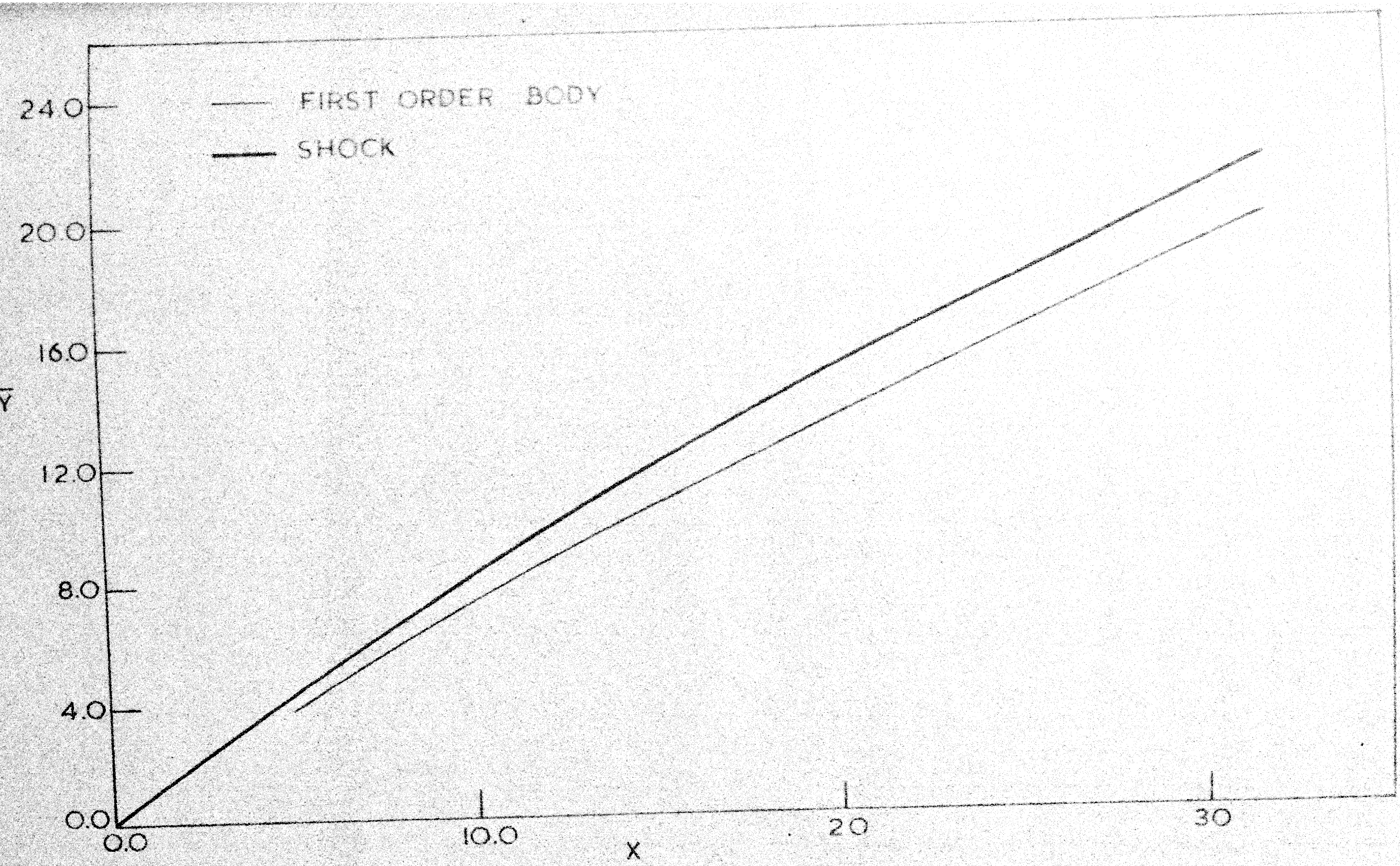

FIG. 6 – BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{9/10}$ WITH $M = \infty$ AND $\gamma = 1.4$

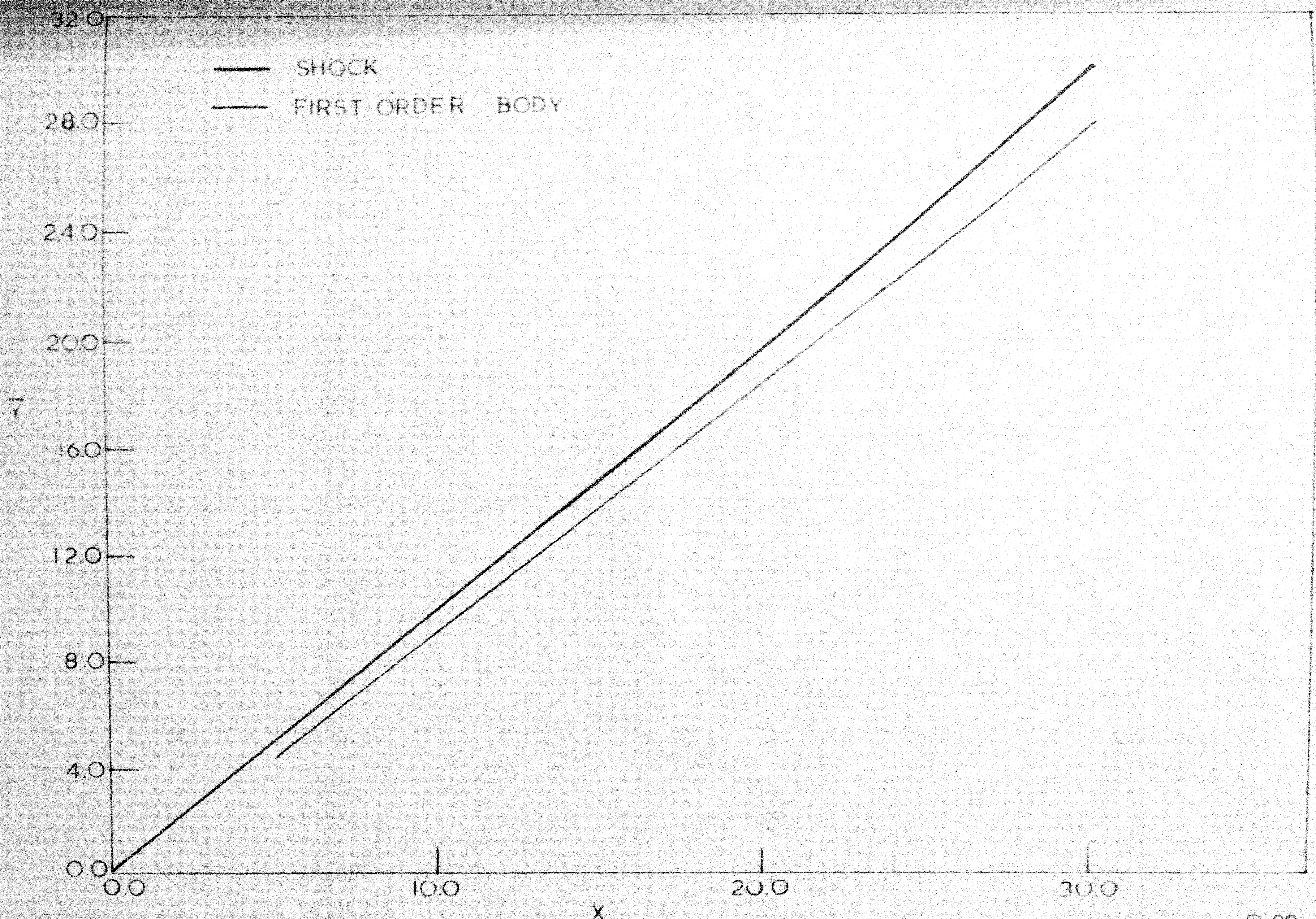

FIG. 7 – BLUNT AXISMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{0.99}$ WITH $M = \infty$ AND $\gamma = 1.4$

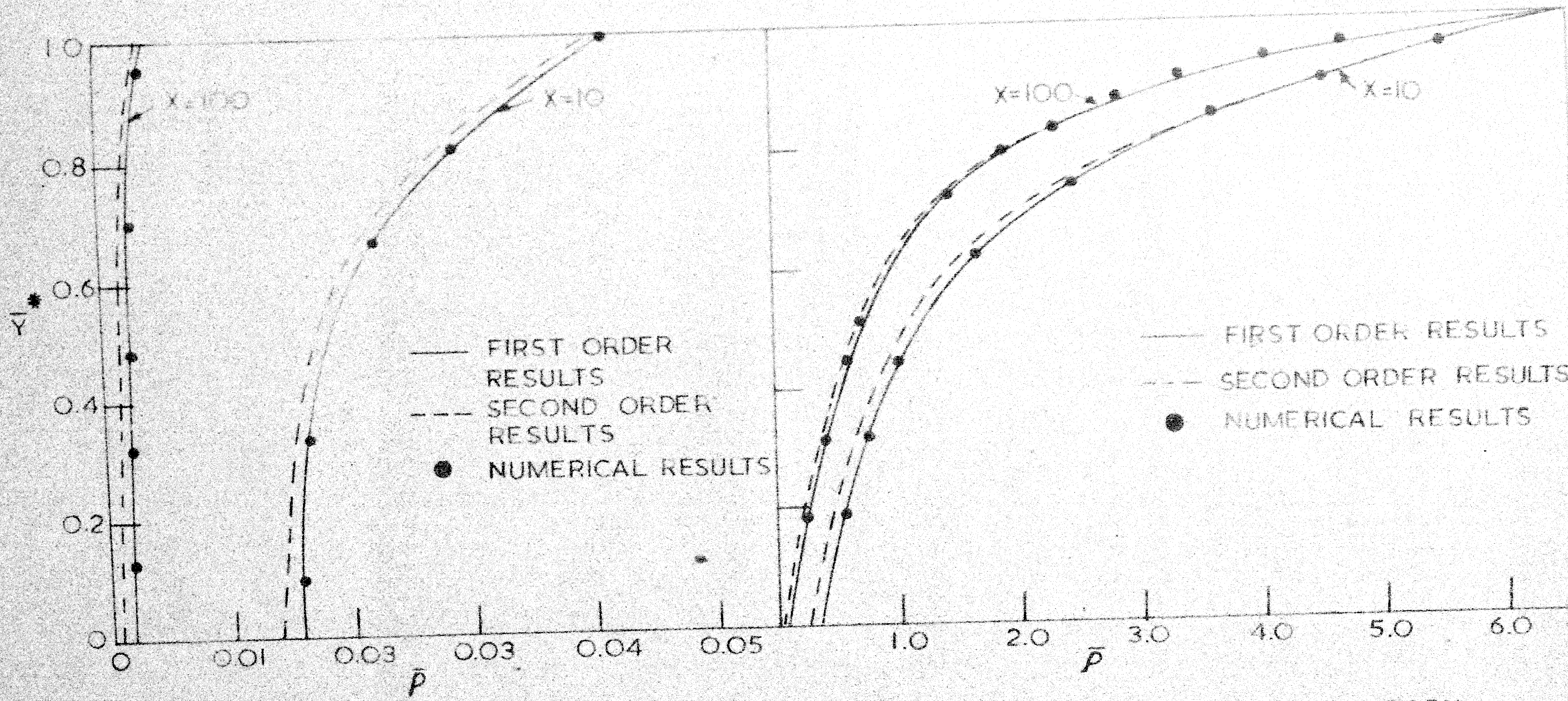

FIG 8 - PRESSURE AND DENSITY PROFILES FOR A BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y=\sqrt{2}\,.\,X^{1/2}$

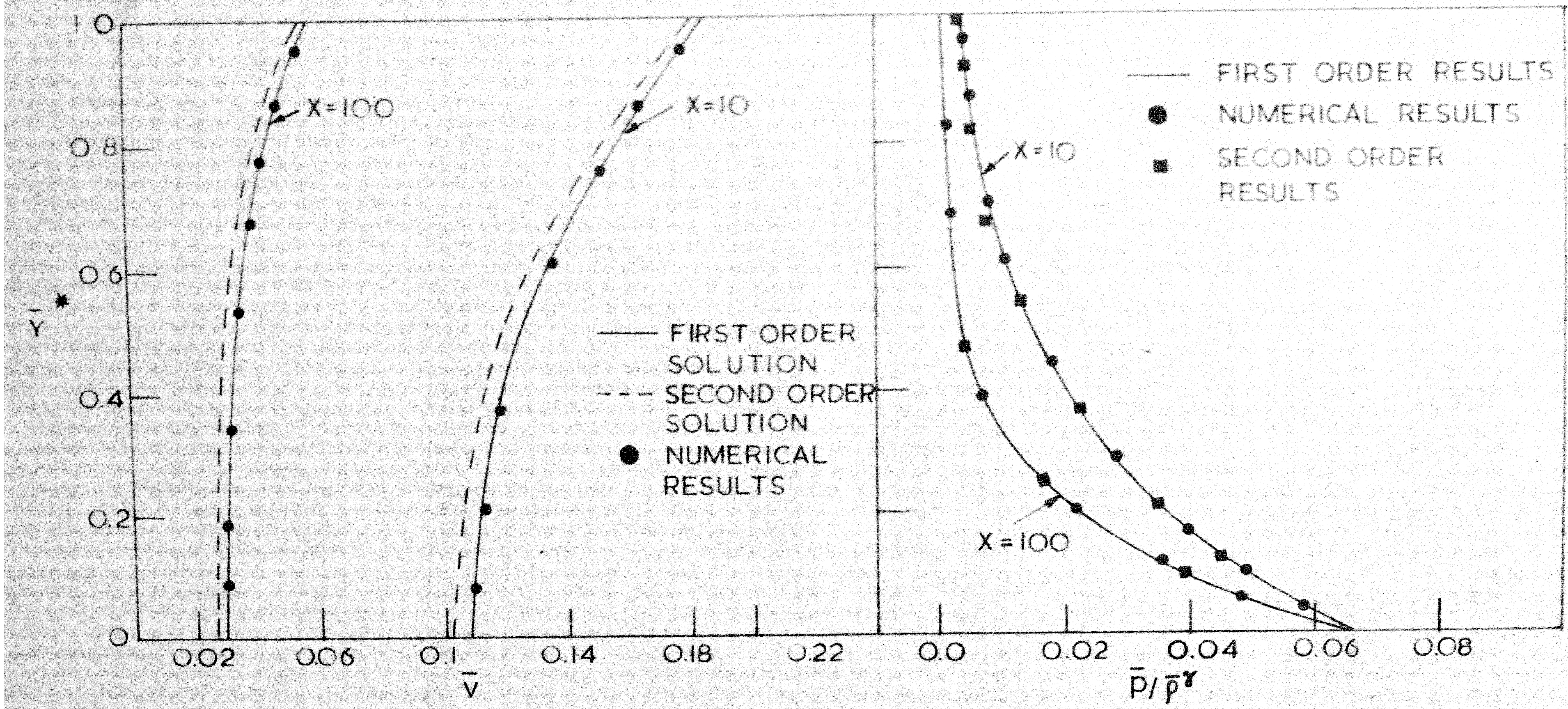

FIG. 9 – 'V' VELOCITY AND P/ρ^{γ} PROFILES FOR A BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y=\sqrt{2}\ X^{1/2}$

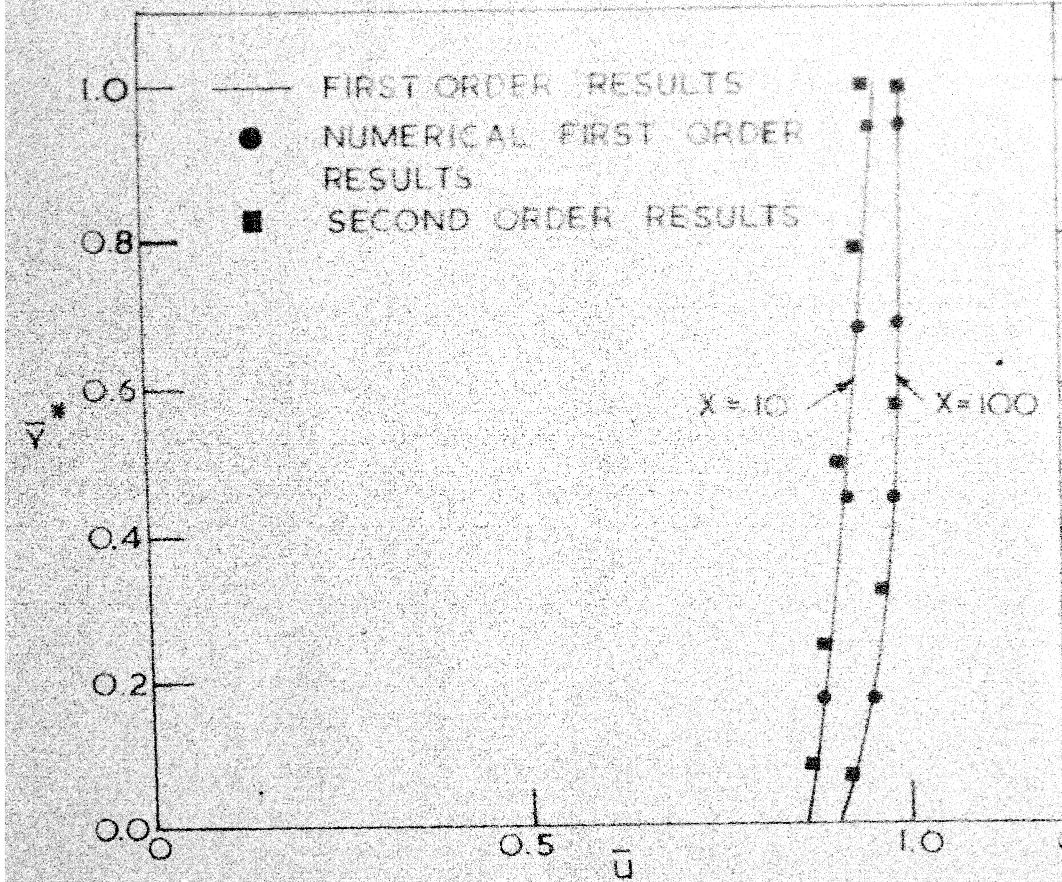

FIG. 10 – 'u' VELOCITY PROFILE FOR A BLUNT AXISYMMETICC BODY PRODUCING A POWER LAW SHOCK $Y=\sqrt{2}\,X^{1/2}$

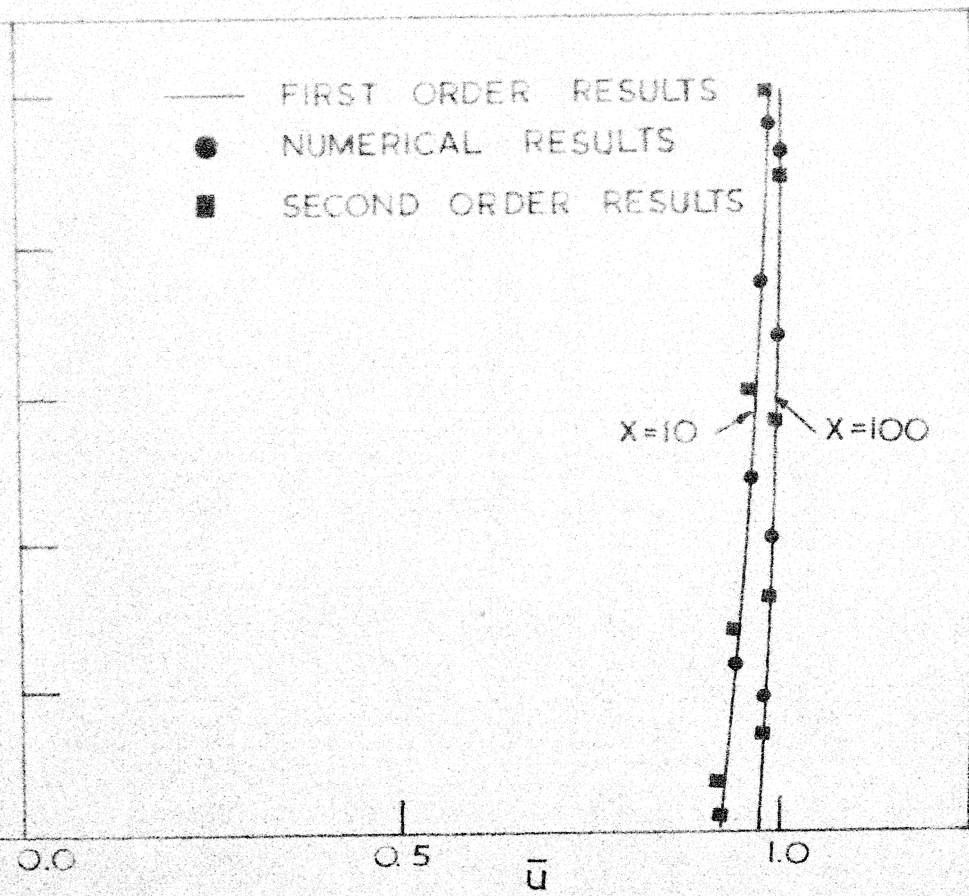

FIG. 11 – 'u' VELOCITY PROFILES FOR A BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$

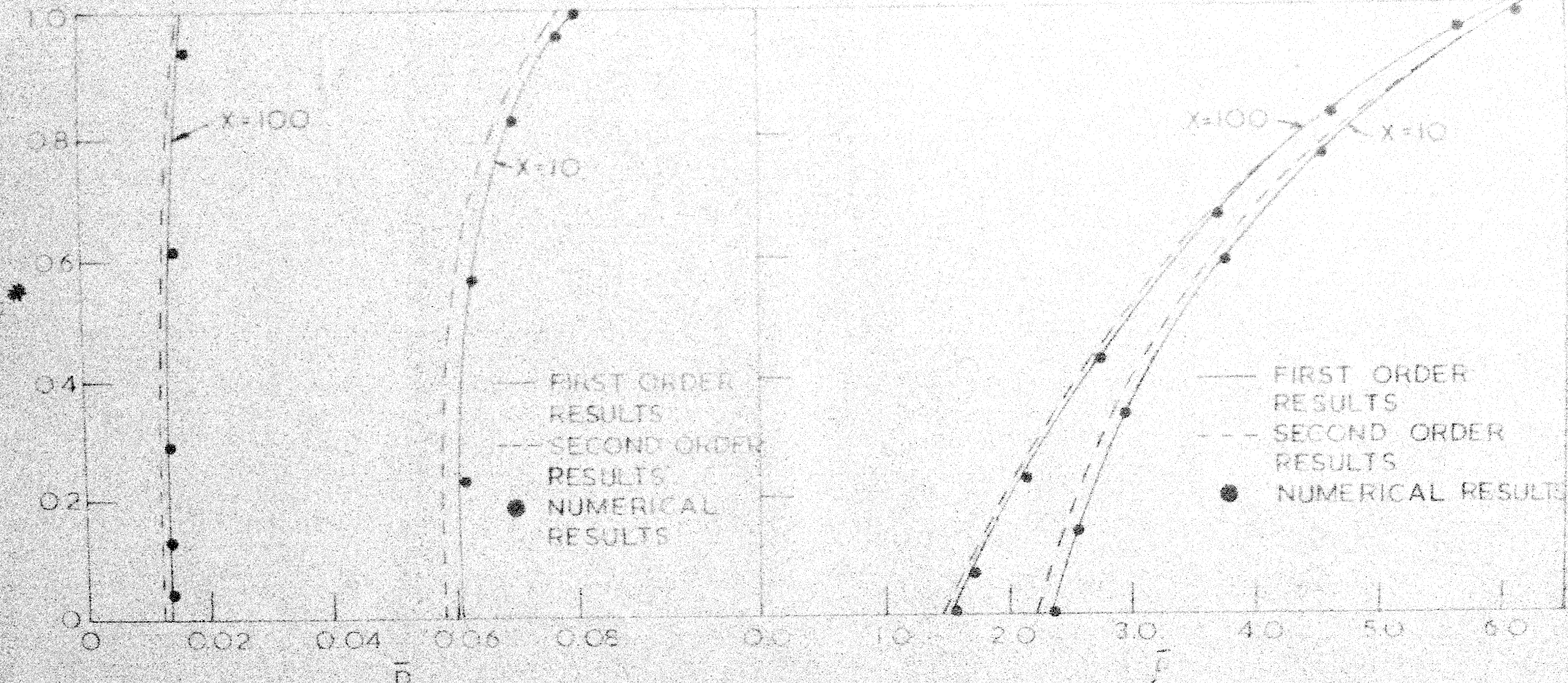

FIG 12_PRESSURE AND DENSITY PROFILES FOR A BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$.

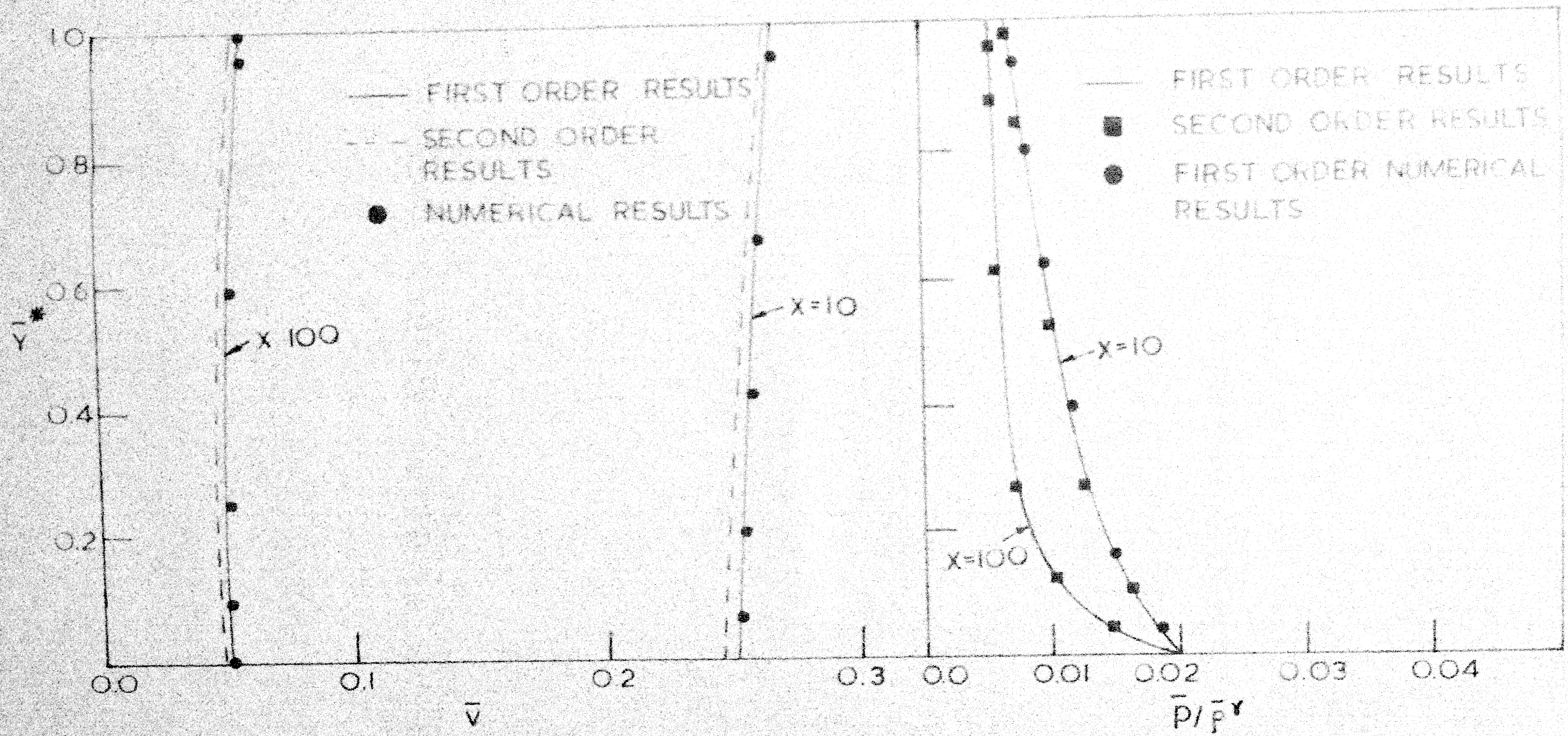

FIG.13_'V' VELOCITY AND $\bar{P}/\bar{\rho}^{\gamma}$ PROFILES FOR A BLUNT AXISYMMETRIC BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$

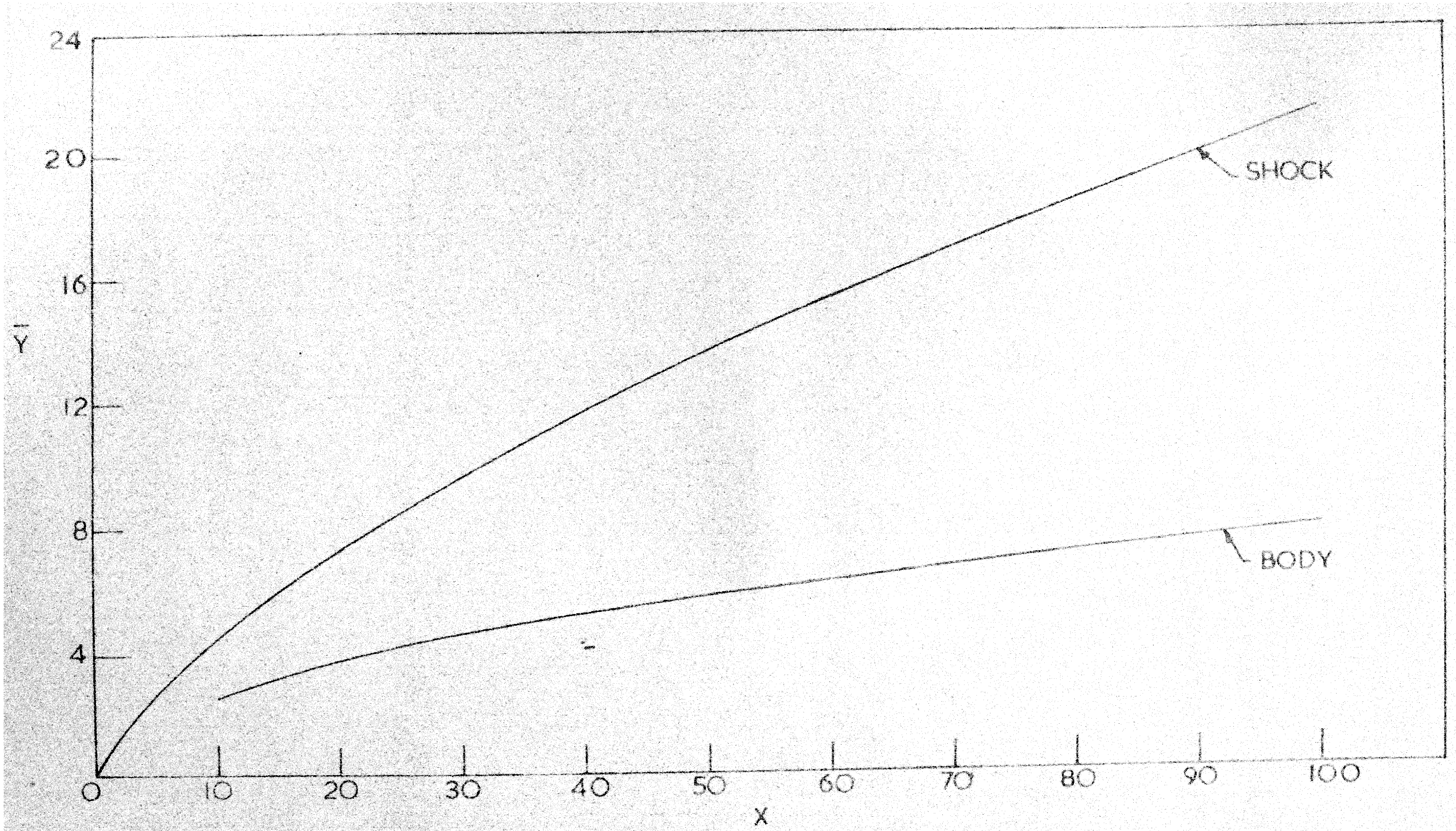

FIG 14 _ BLUNT PLANE BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$ WITH $M = \infty$ & $\gamma = 1.4$

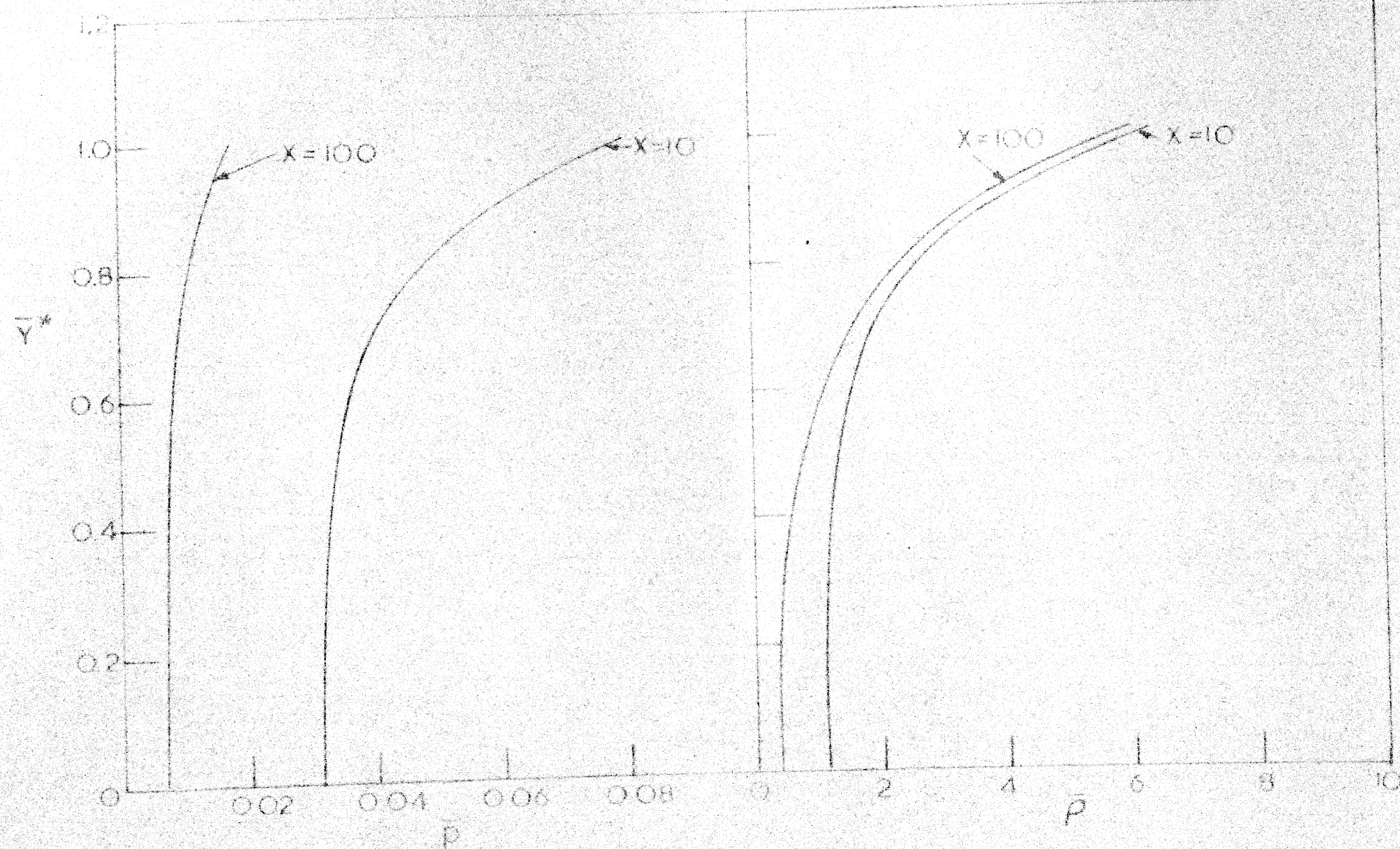

FIG. 15_ PRESSURE AND DENSITY PROFILES FOR A BLUNT PLANE BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$

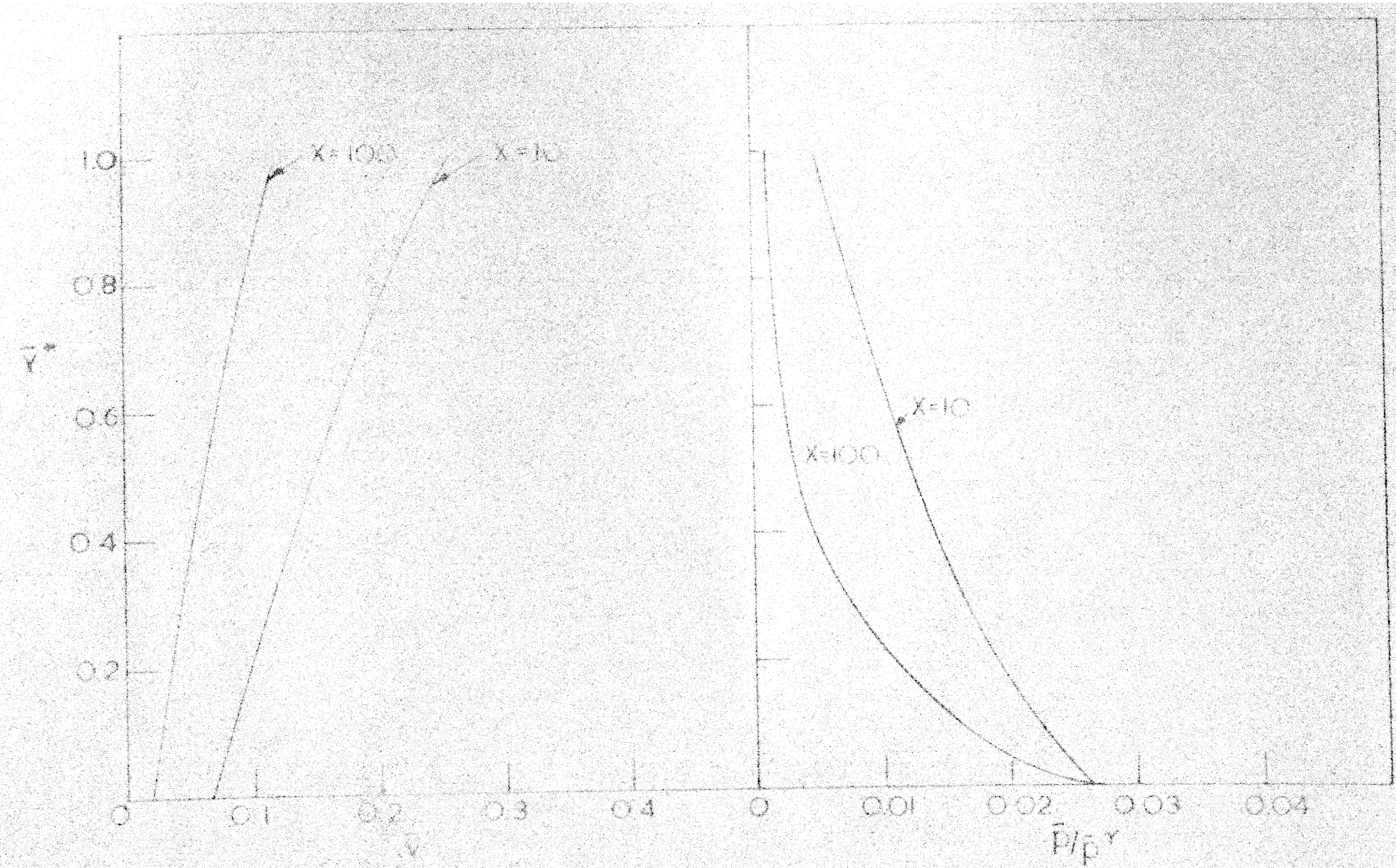

FIG 16. 'V' VELOCITY AND $\bar{p}/\bar{\rho}^{\gamma}$ PROFILES FOR A BLUNT PLANE BODY PRODUCING A POWER LAW SHOCK $Y = X^{2/3}$

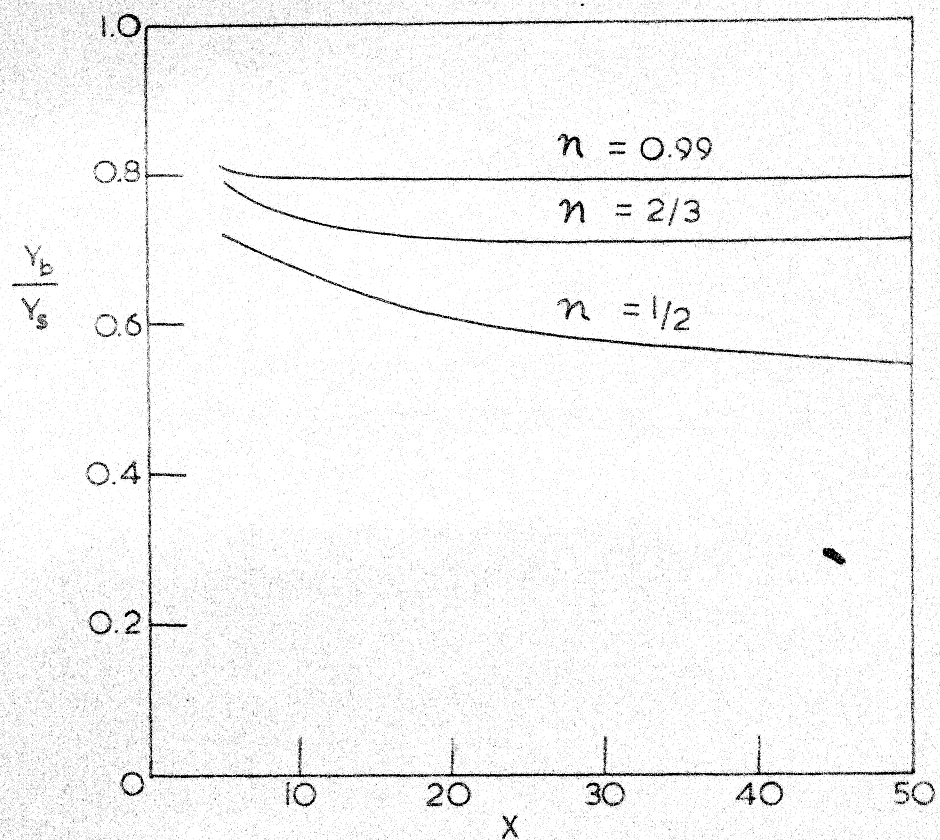

FIG. 17_RATIO OF THE BODY TO THE SHOCK DISTANCE IN AXISYMMETRIC FLOW.